Purchase Order Financing

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June 18nd, 2022

What is Purchase Order Financing?

- Purchase Order Financing is a type of loan.
- First a firm or growing company writes a contract with interested customers saying they will provide a certain volume of goods for a specific price.
  - This contract is called a purchase order.
  - The firm incurs a cost if they can't fulfill the purchase order.
- Then the firm takes the purchase order to a investor who decides what loans the firm can be approved for using the purchase order.

Purchase Order Financing at Work

PlateTopper

- Firm that makes a device that lets you turn a plate into a tupperwear container.
- Struggled to find funding because they hadn't yet developed the techniques for mass production and had no collateral.
- Had a large purchase order contract in place with Walmart (worth $1 million) who wanted to sell PlateToppers in their stores.
- Turned to PO financing to get a loan.
- PO financing loan was much larger than it would have been without the clear show of customer interest.

Key Questions

1. What do the set of purchase order financing and corresponding funding contracts look like?
2. How does purchase order financing compare to funding in an environment where a firm only has access to debt or equity?

Preview of the Main Results

- In the unique Perfect Bayes Equilibrium that satisfies the Intuitive Criterion:
  - Firms with fewer customers under-report their customer interest to the investor.
  - Firms with more customers accurately report.
- Purchase order financing increases the amount of capital a firm can secure.
- Firm's prefer Purchase Order Financing to traditional Debt financing.

Model Overview

- A firm seeks financing from a investor.
  - The firm can write purchase order contracts beforehand.
- Developing the product requires combination of investment from the investor and costly effort from the firm.

Environment

- Firm has customers interested in purchasing $\rho \in [0, \bar{\rho}]$ units of their good for unit price v.
  - ρ is privately known by the firm.
  - v is commonly known.
- Investor has prior belief G about ρ.
  - G is atomless.
  - G has pdf g.

Contracts

Step 1 - Purchase Order Contracts

- The firm selects the number of purchase orders $\hat{\rho} \leq \rho \leq \bar{\rho}$ to commit to delivering in exchange for v.
- If the firm fails to produce and deliver, incurs cost $R > 0$ per unit.
  - R is exogenous.

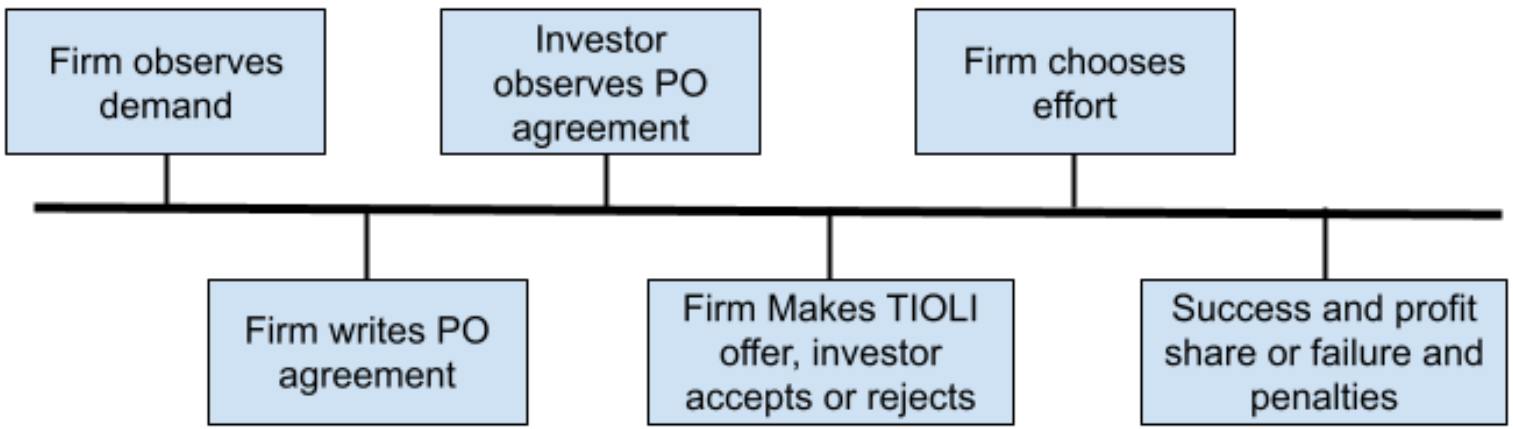
Step 2 - Funding Contracts

- Investor observes $\hat{\rho}$.
- Firm makes a take it or leave it offer to the investor.
- Offer is an initial loan of size $\ell \geq 0$ and a debt repayment $d \geq 0$.
- investor has acceptance rule a which can depend on $(\hat{\rho}, \ell, d)$.
- Debt is only paid back when the firm successfully develops product.

Production

- Given funding contract (ℓ, d) and purchase order contract $\hat{\rho}$, the firm chooses effort $e \in E$.
  - Effort is unobservable to the investor.
  - Effort has cost $c * e$ with $c > 0$.
- Production is binary - success or failure.
  - Product is successfully produced with probability $F(\ell, e) = \beta(e\ell)^{\alpha}$ where $\beta > 0$ and $0 < \alpha < \frac{1}{2}$.

Timeline


Firm observes demand
Investor observes PO agreement
Firm chooses effort
Firm writes PO agreement
Firm Makes TIOLI offer, investor accepts or rejects
Success and profit share or failure and penalties

Payoffs

Payoff to firm:

$$(v\rho - d)F(\ell, e) - (1 - F(\ell, e))R\hat{\rho} - ce \text{ if offer is accepted}$$

$$-R\hat{\rho} \text{ if offer is rejected}$$

Payoff to investor:

$$d * F(\ell, e) - \ell \text{ if offer is accepted}$$

$$0 \text{ if offer is rejected}$$

Perfect Bayes Equilibria that Satisfy the Intuitive Criterion

Basic Solution Concept - Perfect Bayesian Equilibrium

- Investor accepts offers that have weakly positive expected values given their beliefs about the firm's type.
- Firms select the PO agreement/funding contract ask that maximize their expected profit.
  - Subject to the restriction that the investor approves the funding contract given the PO agreement in place.

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Refinement on Allowable Beliefs - Intuitive Criterion

- If the investor sees an "off-path" set of contracts, it wasn't offered by a type of firm who would be better off under their equilibrium contracts than under the off-path contracts.

No Pooling

Lemma

No two different types propose identical funding and purchase order contracts in equilibrium.

- Higher type firms are more willing to take on additional debt for larger initial funding.

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- Higher type firms are more willing to take on additional debt for larger initial funding.
- Implies a virtual Incentive Compatibility constraint on firms in eq:

$$(\hat{\rho}_\rho, \ell_\rho, d_\rho, e_\rho) = \text{argmax}(v\rho - d)F(\ell, e) - (1 - F(\ell, e))R\hat{\rho} - ce$$

$$\text{s.t. (VIC) } (v\rho - d)F(\ell, e) - (1 - F(\ell, e))R\hat{\rho} - ce \geq (v\rho - d_{\rho'})F(\ell_{\rho'}, e) - (1 - F(\ell_{\rho'}, e))R\hat{\rho}_{\rho'} - ce \ \forall \rho' \neq \rho$$

$$\text{(Inv IR) } \mathbb{E}_g[d * F(\ell, e) - \ell] \geq 0.$$

Purchase Orders As A Sufficient Statistic

Key Lemma

In a Perfect Bayes Equilibrium that satisfies the Intuitive Criterion, the Purchase Order Agreement enforces the Incentive Compatibility constraint meaning:

$$(\ell_\rho, d_\rho) = argmax_{\ell,d\geq 0} u_\rho(\hat{\rho}, \ell, d) \text{ s.t. } u_{inv}(\ell, d, e(\rho, \hat{\rho}, \ell, d)) \geq 0.$$

- This means we can derive the optimal equilibrium path of purchase order agreements from the IC constraint and the loan and debt will follow immediately.

Using IC to Derive Optimal Purchase Order Agreements

Let $\tilde{\rho}(\rho)$ be the solution to the following differential equation with initial condition $\tilde{\rho}(0) = 0$:

$$\frac{\partial \tilde{\rho}}{\partial \rho} = \frac{v}{R} * \frac{k_1 * (v\rho + R\tilde{\rho})^{\frac{2\alpha}{1-2\alpha}}}{1 - k_2 * (v\rho + R\tilde{\rho})^{\frac{2\alpha}{1-2\alpha}}}.$$

- $\tilde{\rho}$ is the unique solution with image contained in $\mathbb{R}^+$.
- Notice there exists a $\rho^* > 0$ such that $\tilde{\rho}(\rho^*) = \rho^*$.

Characterization

Proposition

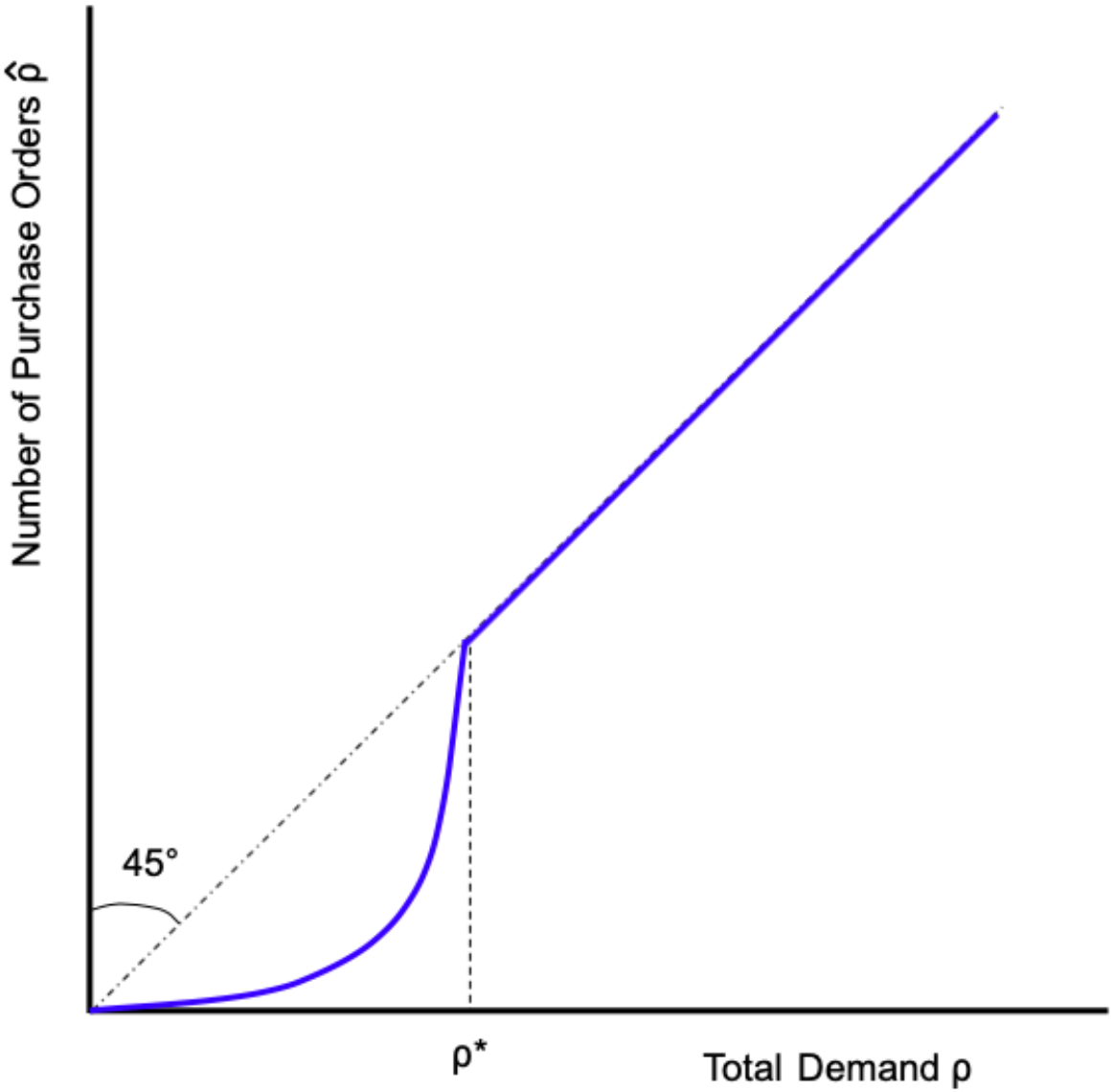
The unique set of contracts allowable in a Perfect Bayes Equilibrium that satisfies the Intuitive Criterion are $(\hat{\rho}_\rho, \ell_\rho(\hat{\rho}_\rho), d_\rho(\hat{\rho}_\rho))_{\rho \in [0,\overline{\rho}]}$ where

$$\hat{\rho}_\rho = \begin{cases} \tilde{\rho}(\rho) & \text{if } \rho \leq \rho^* \\ \rho & \text{if } \rho > \rho^*. \end{cases}$$

$$\ell_\rho(\hat{\rho}) = \textit{constant} \times (v\rho + R\hat{\rho})^{\frac{1}{1-2\alpha}}$$

$$d_\rho(\hat{\rho}) = \alpha(v\rho + R\hat{\rho})$$

The Optimal Purchase Order Contracts



Above and Below the Threshold

Key trade-off - more purchase orders increase funding and commit firms to higher effort, but also open firm up to larger penalties in the case of failure.

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Above the Threshold

- Customers would prefer offering contracts constructed for higher types.
  - $\hat{\rho} \leq \rho$ is binding and guarantees IC binds.
- Risk of penalty is outweighed by advantages of committing to higher effort and receiving larger loans.
  - Effort and loan size have decreasing marginal returns, but are also complements. If one is too low the other matters less.

Comparisons to a Debt Only Environment

Consider a debt only environment where the firm cannot write purchase orders before approaching the investor with a funding offer. The only contracts are (ℓ_ρ, d_ρ) and the firm's utility is just

$$(v\rho - d)F(\ell, e) - ce.$$

- The investor can only infer the firm's type from their funding contract offer (ℓ, d).

The Unique Debt-Only Separating Equilibrium

There is only one fully separating equilibrium in this environment. $d(\rho)$ satisfies the following differential equation

$$\frac{\partial d}{\partial \rho} = \frac{d * v\alpha^2}{(1-\alpha)[d - \alpha * v\rho]}$$

setting $d(0) = 0$ and the corresponding funding level is:

$$\ell = \left[\frac{\alpha\beta}{c^\alpha} d(v\rho - d)^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{1-2\alpha}}.$$

PO Financing Outperforms Debt

Proposition

Each firm ρ weakly prefers their set of contracts under the unique Perfect Bayes Equilibrium that satisfies the Intuitive Criterion to their set of contracts under the unique separating debt-only equilibrium.

- Purchase order financing gives the firm another dimension to signal type to investor and create separation.
- Allows for a more favorable ratio of debt and loan size.
  - The ratio of loan size to debt $(\ell_\rho : d_\rho)$ is larger under purchase order financing than debt-only financing.

A Quick Note on Equity Contracts

In this environment, both agents are risk neutral so debt-contracts will outperform equity and preferred equity contracts (see Jensen and Meckling (1976)). Therefore purchase order financing outperforming debt implies that it also outperforms equity contracts.

- This conforms with what we know about companies that use purchase order financing,
  - Most of these companies are reasonably small.
  - Don't have small probabilities of being highly successful so investors are not interested in issuing equity contracts.

Conclusion

- Characterized the unique Perfect Bayes Equilibrium that survives the Intuitive Criterion in a Purchase Order environment
- Highlighted the bifurcated nature of this fully separating equilibrium:
  - Firms with fewer customers under-report their customer interest to the investor to avoid excessive penalties from failing to fill Purchase Orders.
  - Firms with more customers fully report.
- Showed that firm's prefer Purchase Order Financing to Traditional Debt financing and Equity Financing.

Thank you!