Purchase Order Financing: A Signalling Approach

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August 10, 2022

Abstract

This paper proposes a model of purchase order financing, a technique for funding firms with low collateral via debt in which investors consider agreements with customers to purchase finished goods from the firm, called purchase order agreements, when deciding whether a firm's funding proposal is acceptable. I identify a unique Weak Perfect Bayesian Equilibrium that satisfies the Intuitive Criterion. This equilibrium is fully separating, meaning that the firm proposes a unique and increasing loan size and debt repayment quantity based on their level of demand from customers. Moreover firms with large amounts of customer demand write purchase order agreements which reflect the full extent of their demand in order to demonstrate to investors their potential profitability and commit themselves to higher levels of effort, while firms with less demand, 'shade down' the level of demand written into their purchase order agreements in order to avoid larger penalties should they fail at production. I compare this setting to a setting where firms can only access debt to demonstrate how firms with little collateral turn to purchase order financing over more traditional debt financing because purchase order financing allows the firms to acquire larger loans. The terms for loans are more favorable with purchase order financing because purchase orders serve as better signals of the firm's demand and help alleviate moral hazard by committing firms to higher levels of effort.

1 Introduction

Purchase order financing is a type of loan that extends credit to firms with little capital but an interested consumer base. The key building block of purchase order financing is a Purchase Order Agreement. Purchase Order Agreements are contracts written between a firm and a customer (usually another business) interested in a product which the firm has not yet created. The Purchase Order Agreement outlines precisely which products or services the firm will deliver to customer. It describes quantities, deadlines, the price the customer will pay. The firm will incur penalties for failure to deliver its product on time. Importantly, the customer does not pay the firm up front; it only pays the agreed upon price after receiving the firm's product. This means that the firm will have to find alternative sources of funding to initiate production.

Purchase Order Financing is when a firm uses a Purchase Order Agreement from a credit-worthy customer as a verifiable signal of demand and investors use the agreement to secure dramatically larger loans than the firm could otherwise get approved.[1] While purchase order agreements are widespread in business to business transactions, purchase order financing is only involved in a subset of these agreements.[2]

The goal of this paper is to determine why we see firms who lack adequate collateral turn to purchase order financing in lieu of more traditional forms of financing, for example pure debt financing. First I characterize equilibrium contracts in a model of purchase order financing, then I compare these contracts to equilibrium contracts in an environment where the firm only has access to traditional debt financing.

Consider a firm who is developing a new product. The firm is capital constrained and requires an outside investor to fund development and production of the product. The prob-

[1] Purchase order financing has much worse interest rates than traditional debt, however this stems from the fact that most firms engaging in purchase order financing can't obtain sufficient loans to begin with (Martin 2010). Although traditional debt may seem superior to purchase order financing, the terms that firms who turn to purchase order financing would receive under debt financing would be inferior.

[2] Purchase orders are often used as a mechanism for businesses to keep track of their manufacturing supply chains.

ability that the firm will successfully produce is increasing in funding level effort exerted by the firm. The firm has a customer who would like to purchase the good and has private information about what quantity that customer is interested in. The firm selects what fraction of that quantity they commit to selling using a purchase order agreement. If the firm fails to deliver on this agreement, they incur a reputational penalty proportional to the size of the purchase order. The firm then brings their purchase order agreement to an investor to whom they propose a debt contract with limited liability. The purchase order signifies to the investor a credible lower limit on the firm's demand. If the investor agrees to the offer, the firm chooses a level of costly effort. Production success is binary and is an increasing and concave with respect to both effort and investment level. If production is successful, the investor's debt gets repaid. When production is unsuccessful, the firm cannot deliver on their purchase order and incurs a reputational cost, however due to limited liability the investor absorbs the loss on their investment.

The equilibrium concept used in this paper is a unique Weak Perfect Bayesian Equilibrium that satisfies the Intuitive Criterion. I use the intuitive criterion because it reduces the number of equilibrium while only imposing minimal restrictions on the investors beliefs - namely that deviations from equilibrium contracts would not be proposed by firms who were better off in equilibrium than they would be under the out-of-equilibrium contract (Cho and Kreps, 1987). This model predicts a unique Weak Perfect Bayesian Equilibrium that satisfies the Intuitive Criterion. This equilibrium is fully separating, meaning that for every level of demand a firm has, it proposes a different financing contract to the investor. Moreover, the loan size and level of debt a firm proposes to the investor is strictly increasing in the firm's level of demand. The equilibrium takes on a bifurcated structure where firms with low customer demand "shade down" the quantity stipulated in their purchase orders in order to avoid higher penalties should they fail to successfully produce their product. Firms with higher levels of demand are incentivized to exert higher levels of effort and are also approved for larger loans, and thus are more likely to be successful in production. As such, firms with higher levels of demand write the largest purchase order contracts they are able to since the additional loans they can acquire with larger volume orders outweigh the potential penalty they incur should they fail to produce.

Next, this paper compares the equilibrium where the firm has access to purchase order financing to equilibria in which the firm only has access to traditional debt financing. In comparison to the purchase order equilibrium, under the Debt-Only equilibrium, the firm gets approved for smaller loans with worse repayment rates. This is because purchase orders lead to larger levels of effort. This tracks with interviews from small sized firms using purchase order financing who reported that they appreciated having access to this type of credit as it was the only way they'd found to get approved for loans large enough to fund production (Martin 2010).

As an extension this paper considers the case where the firm faces a monetary penalty stipulated to in the purchase order agreement instead of an exogenous reputational penalty should they fail to deliver on the purchase orders. In this case, firms do not "shade down" the number of purchase orders they write. Instead all firms, regardless of demand write purchase orders covering the full volume of goods their customer is interested in. This is because now the firm has some freedom in the level of penalty they incur with the customer. Thus they use the quantity specified in the agreement to perfectly signal their demand to investors, and set the penalty in order to commit themselves in advance to the level of effort that optimizes their ex-ante profits.

The rest of this paper reviews the literature Section 1.1 and gives a formal description of the model (Section 2). Section 3 lays out the equilibrium concept used in this paper, a Weak Perfect Bayes Equilibrium satisfying the Intuitive Criterion, and then solves for the unique such equilibrium. Section 4 compares the purchase order financing setting to one where only traditional debt financing is available. Section 5 discusses an extension where the firm's non-delivery penalty is monetary and endogenous and Section 6 concludes.

1.1 Related Literature

This paper is closely related to the capital markets literature, which concerns the optimal way to for firms to secure funding from investors. This strand of literature begins with Modigliani and Miller (1958) who argue that a firm's capital structure does not affect the firm's value. Leland and Pyle (1977), Ross (1977), and Myers and Majluf (1984) introduce private information for the firm regarding the firm's quality. All three papers argue that the firm's funding request from an investor will serve both to help the firm succeed, and to signal to an investor some indication of the firm's quality. This paper introduces purchase order financing as both a way for the firm to commit itself to higher effort, and a signal to the investor of the firm's level of demand, i.e. potential for profitability.

Brennan and Kraus (1987) and Noe (1988) both highlight that as the set of tools investors use to fund firms increases, there is a richer signal space, and firms can use new capital structures that simultaneously serve as good signals and impose a lower cost on the firm than the structures they were using before. This is related to the comparison of traditional debt financing to purchase order financing; I find that firms prefer purchase order financing, in part because they have an additional degree of freedom with which they can signal their type to investors. This allows them to obtain more favorable debt terms because they aren't using the loan size and debt repayment as indicators of their type and can better optimize those portions of the financing agreement.

While the above papers address the problem of adverse selection, however there is a parallel literature on moral hazard (started by Jensen and Meckling, 1976). These papers consider how different capital structures change firm's incentives to exert effort and find that capital structures that expose firms to the downside risks of failing to exert effort are the structures best at alleviating moral hazard concerns. This paper finds that purchase order financing helps alleviate moral hazard to a larger extent than traditional debt financing because it introduces the penalty for failure into the firm's utility.

Noe and Rebello (1996) and Fulghieri and Lukin (2001) both address capital structure decisions under both moral hazard and adverse selection. Both papers predict that firms with low growth potential are more likely to rely on forms of debt rather than other types of financing. Firms who rely on purchase order financing usually have little potential for large growth, so these findings help motivate why this paper uses traditional debt financing as the basis of comparison for purchase order financing as opposed to other capital structures.

Companies contracting with one another are far more willing to extend forms of assistance which are not loans, due to the moral hazard problem associated with handing over cash (Burkart and Ellingsen, 2004). Reindorp, Tanrisever, and Lange 2018 explain that this is the primary reason that a customer is willing to enter into an agreement with the firm that they will pay upon delivery, but are not willing to front the cash themselves. On the other hand, lenders specialize in assessing the risk of loaning money to firms and are much better equipped to monitor the firm. Taken together, these two findings suggest a role for purchase order financing, as opposed to customers directly financing product development through prepayment or other means. Huang, Wu, and Chiang (2014) point out that purchase order agreements signed with credit-worthy customers make the investor much more willing to finance the firm, since this significantly reduces risk to investors by showing a clear minimum level of demand.[3] However the authors point out that there is still some risk stemming from uncertainty that the firm will successfully produce the goods contracted. This tracks closely with this paper's finding that purchase order financing plays a role in mitigating the adverse selection problem between firms and investors, but does also alleviate the moral hazard problem to some extent.

Finally, this paper relates to the quality disclosure literature starting with Grossman (1981). This literature studies the incentives for a firm to release verifiable signals of their own quality to potential buyers. Many insights can be applied to firms using purchase orders as verifiable signals to potential investor's about the firm's potential profitability.

[3] There is also a Purchase Order Financing literature within Operations Research literature which is studied in the context of stabilizing customer supply chains (Reindorp, Tanrisever, and Lange 2018; Tang, Yang, and Wu 2018; Wu, Huang, and Chiang 2014; Wu 2017). These papers focus on the interaction between the customer and the firm whereas this paper explores the interplay between the firm and the investor.

Grossman's 'unravelling' result shows that first, the highest quality firms disclose their type to distinguish themselves from lower types, then the highest of the remaining types will deviate to avoid being pooled with the remaining lower types. This logic proceeds until all types have disclosed which generates full separation. The unravelling logic is the key to this paper's result that any weak perfect bayesian equilibrium that satisfies the intuitive criterion will have full separation of types as any potential pooling group will unravel. The canonical papers on disclosure assume that disclosing information is costless (Grossman (1981), Milgrom (1981)), however later work showed that when disclosing is costly, firm's above a certain quality threshold reveal their type and those below the threshold do not (Grossman and Hart (1980), Jovanovic (1982)). Here the cost of disclosing one's type is not binary; writing purchase orders is associated with increasing per unit costs. Hence above a threshold firm's 'fully disclose' (i.e. write purchase orders capturing all their demand) while firms below the threshold will under-report their type - committing to just enough cost to generate the unravelling.

2 Model

There are three parties; the firm, the investor, and the customers interested in the firm's product. The firm has a mass $\rho \in [0, \overline{\rho}]$ of customers interested in the product the firm is developing where $\overline{\rho} > 0$. Each customer has willingness to pay $v > 0$ for the firm's final product. The mass of customers ρ is the firm's private information. The investor has a prior belief G about the value of ρ where G is atomless and has pdf g. The demand ρ will determine the total revenue for the firm.

After the firm gets a signal of their customer's demand, ρ, the firm may write purchase order agreements with the customer. Firms with demand, ρ, (e.g. type ρ) can choose volume $\hat{\rho} \in [0, \rho]$ of goods to promise to the customer in the form of a purchase order agreement. This requires that the firm cannot write more purchase orders than it has customer demand which is expected since a purchase order agreement is written with the consent of the customer, and so the firm cannot inflate the demand represented by purchase order agreements.

Now suppose the firm has all the bargaining power[4] and thus can set price v for the product. Then each purchase order contract specifies that the firm will charge price v to the customer upon receipt of the final good, and should the firm fail to deliver the good, they incur penalty a per unit penalty $R > 0$. Assume R is exogenous. It can be thought of as either legal costs associated with failing to deliver the order, or a repuatational cost to the firm.[5] The total penalty to the firm incurred by failing to deliver on a purchase order agreement is increasing in the size of that agreement, $\hat{\rho}$, as larger contract breaches would lead to larger lawsuits and more reputational repercussions.

To be able to produce the good the firm must write a funding contract with the investor. Recall that the firm is liquidity constrained and their only source of funding is the investor. The firm shows the investors their purchase order agreements $\hat{\rho}$ and makes a take it or leave it offer to the investor which specifies a loan of size $\ell \geq 0$ and a debt repayment $d \geq 0$. Assume for simplicity that under the funding contract the firm pays back debt d when they successfully produce their good, and pay back nothing otherwise. After observing the purchase order agreements and the firms funding proposal, the investor form's posterior belief h over the firm's type and either accepts or rejects the funding proposal.

Once the funding contract is signed the firm selects effort level $e \in [0, 1]$ towards product development. The firm incurs cost of effort $c * e$ where $c > 0$.

Production proceeds as follows. The probability that the product can be successfully mass produced is a function of both funding and effort $f(e, \ell) = \min\{1, \beta(e * \ell)^{\alpha}\}$ with $\beta > 0, \frac{1}{2} > \alpha > 0$. Note that effort and funding level are complimentary but both have diminishing marginal returns.

[4]This is a reasonable assumption if the firm has created a new product and is a monopolist. Moreover both Wu (2017) and Reindorp et. al. (2018) argue that customers should be very 'generous' on pricing terms in purchase order agreements being used for financing to maximize the chance that the firm will secure a favorable loan and successfully deliver the product.

[5]For a discussion of a model where R is a monetary fine paid to the customer and written into the purchase order agreement see Section 5.1.

If the product is successfully developed, the firm delivers the good to all customers he wrote contracts with as well as all customers he didn't write them with[6]. Then the firm receives payment v from the full mass ρ of customers interested in his product. After receiving $v\rho$ from the customers, the firm pays back his debt, d, to the investor.

If the product can not be successfully developed and produced at scale, then the firm must renege on all purchase orders he wrote. If the firm wrote $\hat{\rho}$ purchase orders, then he incurs penalty $R\hat{\rho}$ associated with reneging where $R \leq v$. There is limited liability on the part of the firm so the investor does not receive any repayment on his loan.

The following condition on the cost of effort ensures that there will be no corner solutions on effort and that for any type ρ the firm's optimal effort is not such that they put in sufficient effort to produce with probability 1.

$$c^{\alpha} \geq \alpha\beta^{\frac{1+2\alpha}{2}}(1-\alpha)^{\alpha}2v\overline{\rho} \tag{1}$$

The condition can be interpreted as effort must be sufficiently costly vis-a-vis the probability of success (as measured through α and β) and the profits from success (as measured by v) that even the highest type firm will not exert full effort.

The payoffs of a firm of type ρ who has $\hat{\rho}$ purchase orders, funding contract (ℓ, d) and exerts effort e is total revenue, minus the penalty for reneging on purchase orders times the number of orders times the probability that production fails, minus the debt repayment when production is successful with the caveat that the firm cannot pay back more than their total revenue. From this quantity, subtract the linear effort cost. The firm's utility is thus

$$\hat{u}_{\rho}(\hat{\rho}, \ell, d, e) = (v\rho - d)f(e, \ell) - R\hat{\rho}(1 - f(e, \ell)) - ce \tag{2}$$

[6]Note that if some of the customers who aren't locked in via purchase order disappear, then purchase order contracting becomes more appealing to the firm and the entrepreneur. This paper is exploring how Purchase Order Financing can alleviate moral hazard and adverse selection problems, so I disregard this additional and clearly advantageous aspect of Purchase Order Financing

Utility to the investor is the debt repayment they receive when the firm's production is successful (note it cannot exceed the firm's total revenue since the firm has limited liability) minus the loan they initially gave the firm. Mathematically it is:

$$\hat{u}_I(\rho, \ell, d, e) = (\max\{d, v\rho\})f(e, \ell) - \ell \tag{3}$$

3 Equilibrium

This section defines the solution concept, a Weak Perfect Bayesian Equilibrium that satisfies the intuitive criterion, and then characterizes the unique such equilibrium. To simplify the analysis, we first solve for the effort strategy of a firm of type ρ present in any Subgame Perfect Equilibrium and redefine payoffs to encapsulate this strategy.

At the point where the firm determines their effort level, all contracts are already written so effort becomes a simple maximization problem. The firm chooses effort to solve:

$$\max_{e \geq 0} \; (v\rho - d)\beta(e\ell)^\alpha - R\hat{\rho}(1 - \beta(e\ell)^\alpha) - ce. \tag{4}$$

Then for fixed contracts $\{\hat{\rho}, \ell, d\}$ a firm of type ρ selects effort:

$$e_\rho(\hat{\rho}, \ell, d) = [(v\rho - d + R\hat{\rho})\alpha\beta\ell^\alpha/c]^{1/1-\alpha}. \tag{5}$$

This means that both the firm's type and the number of purchase orders they write are going to effect the investor's expected utility through the effect these factors have on the firm's effort. Since effort is strictly increasing in these quantities, so too will be the investor's utility. Now rewrite both player's utilities in terms of type and contract alone.

$$u_\rho(\hat{\rho}, \ell, d) = \hat{u}_\rho(\hat{\rho}, \ell, d, e_\rho(\hat{\rho}, \ell, d)). \tag{6}$$

$$u_I(\rho, \hat{\rho}, \ell, d) = \hat{u}_I(\rho, \ell, d, e_\rho(\hat{\rho}, \ell, d)). \quad (7)$$

3.1 Equilibrium Solution Concept

First I define a Weak Perfect Bayesian Equilibrium which is the basic equilibrium concept used in this paper. A Weak Perfect Bayesian Equilibrium requires that all strategies maximize player's expected payoffs given their beliefs, and that those beliefs are consistent with Bayes Rule along the path of play.

Definition 3.1. *A* ***Weak Perfect Bayesian Equilibrium*** *(WPBE) is a type dependent contract writing strategy* $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho \in [0, \overline{\rho}]}$*, posterior beliefs for the investor,* h*, and an acceptance rule for the investor* a *such that:*

1. *The investor's acceptance rule is to accept a set of contracts whenever their expected value given* h *is weakly positive and reject otherwise.*

$$a(\hat{\rho}, \ell, d) = \begin{cases} 1 & \text{if } \int_0^{\overline{\rho}} u_I(\rho, \hat{\rho}, \ell, d) h(\rho|\hat{\rho}, \ell, d) \partial\rho \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

2. *The investor's posterior belief,* $\tilde{h}$*, updates* h *via Bayes Rule along the path of play.*

$$\tilde{h}(\rho|\hat{\rho}, \ell, d) = \frac{h(\rho) \mathbb{1}_{\{(\hat{\rho}_\rho, \ell_\rho, d_\rho) = (\hat{\rho}, \ell, d)\}}}{\int_{\hat{\rho}}^{\overline{\rho}} h(\rho') \mathbb{1}_{\{(\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'}) = (\hat{\rho}, \ell, d)\}} \partial\rho'}$$

3. *The firm's contract writing strategy maximizes the firm's expected payoff given the investor's acceptance rule:*

$$\hat{\rho}_\rho, \ell_\rho, d_\rho = argmax_{\hat{\rho} \leq \rho, \ell \geq 0, d \geq 0} u_\rho(\hat{\rho}, \ell, d) * a(\hat{\rho}, \ell, d) - R\hat{\rho}(1 - a(\hat{\rho}, \ell, d))$$

This paper uses the Intuitive Criterion, an equilibrium refinement first proposed by Cho and Kreps (1987, [3]) to identify a unique WPBE by restricting beliefs the investor can hold after a purchase order contract that is off the path of equilibrium play.

Definition 3.2. *A WPBE* $\{\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho\in[0,\overline{\rho}]}, h, a\}$ *satisfies the* ***intuitive criterion*** *if the following hold:*

1. *After seeing an on path set of contracts* $(\hat{\rho}, \ell, d)$*, the investor uses Bayes rule on prior* g *to obtain their posterior* h*.*

2. *If the contract* $(\hat{\rho}, \ell, d)$ *is off path, and the the set of types* $P = \{\rho | u_\rho(\hat{\rho}, \ell, d) \geq u_\rho(\hat{\rho}_\rho, \ell_\rho, d_\rho)\} \cap \{\rho \geq \hat{\rho}\}$ *is non-empty, then following* $(\hat{\rho}, \ell, d)$ *the investor's posterior,* h*, should have* $supp(h) \subset P$*.*

Here P is the set of types who both prefer this off path contract to their equilibrium contracts, and can feasible have written the proposed contract because they are capable of writing $\hat{\rho}$ purchase orders. The intuitive criterion says that if the investor sees an off path contract offer, they should assume that said offer came from a type in P because the offer wouldn't have come from a type who either couldn't have written that many purchase order agreements, or would have preferred their contract in equilibrium to this new contract.

Clearly there are still many beliefs the investor can hold following an off-path signal that satisfy the intuitive criterion. Having too many such beliefs could complicate the ultimate goal of this section, identifying the unique set of contracts that arise in an intuitive WPBE. Fortunately, we can exploit the monotonicity of the setting to find a single "maximally pessimistic" belief for the investor. This "maximally pessimistic" belief will have the following property: if a set of contracts is part of an intuitive WPBE for some permissable beliefs on the part of the investor, those contracts will also be part of an intuitive WPBE where the investor has the "maximally pessimistic" beliefs.

Definition 3.3. *Consider the contract writing strategy* $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho\in[0,\overline{\rho}]}$*. The investor's corresponding* ***maximally pessimistic*** *posterior beliefs,* $\tilde{g}$*, that satisfy the intuitive criterion are:*

1. *If $(\hat{\rho}, \ell, d)$ is on the path of play then $\tilde{g}$ is a Bayesian update of the investor's prior belief, g, following an on path purchase order contract, $\hat{\rho}$.*

$$\tilde{g}(\rho|\hat{\rho}, \ell, d) = \frac{h(\rho)\mathbb{1}_{\{(\hat{\rho}_\rho, \ell_\rho, d_\rho)=(\hat{\rho}, \ell, d)\}}}{\int_{\hat{\rho}}^{\overline{\rho}} h(\rho')\mathbb{1}_{\{(\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'})=(\hat{\rho}, \ell, d)\}}\partial\rho'}$$

2. *If $(\hat{\rho}, \ell, d)$ is not on the path of play but the set of types P is non-empty, then the investor believes with probability 1 that the firm is of the lowest possible type who prefers this contract to their equilibrium contract, and could feasibly write this contract:*

$$\tilde{g}(\rho|\hat{\rho}, \ell, d) = \delta_{inf(P)}.$$

3. *If $(\hat{\rho}, \ell, d)$ is not on the path of play and the set P is empty $\tilde{g}^*(\rho|\hat{\rho}, \ell, d) = \delta_{\hat{\rho}}$.*

Here "maximally pessimistic" means that the investor's belief about the firm's type is as low as is possible while still conforming with the intuitive criterion. This means that to check if a set of contracts can be part of an intuitive WPBE, it suffices to check if there are any profitable deviations from that set of contracts under the investor's maximally pessimistic beliefs.

From here I show that all contracts that are part of a WPBE can be written as contracts that are part of a WPBE with maximally pessimistic beliefs.

The "maximally pessimistic" investor beliefs make the investor think an off path contract was offered by a firm with the lowest possible type allowable under the intuitive criterion. If a contract survives under more 'optimistic' beliefs for the investor which also satisfy the intuitive criterion, then any potential profitable deviation is rejected by the that optimistic belief. This means the same potential profitable deviation is by the investor under the maximally pessimistic beliefs because the investor's expectation over the type of firm and thus over their own expected profit will be lower. This logic holds because the equilibrium effort function, $e_\rho(\hat{\rho}, \ell, d)$ is increasing in the firm's type, and thus the investor's utility is increasing in the firm's type. Since all potential profitable deviations are still rejected under the investor's maximally pessimistic beliefs, $\tilde{g}$ the contracts are still part of an intuitive WPBE under $\tilde{g}$.

Lemma 3.1. *Suppose that $\{\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho \in [0,\overline{\rho}]}, h, a\}$ is a WPBE that satisfies the intuitive criterion. Let $\tilde{g}$ be the investor's maximally pessimistic beliefs given $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho \in [0,\overline{\rho}]}$ and let $\tilde{a}$ be the corresponding acceptance rule that satisfies condition 1 of Definition 3 given $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho \in [0,\overline{\rho}]}$ and $\tilde{g}$. Then $\{\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho \in [0,\overline{\rho}]}, \tilde{g}, \tilde{a}\}$ is also a WPBE that satisfies the intuitive criterion.*

Since to check if a set of contracts can be part of an intuitive WPBE, it suffices to check those contracts for profitable deviations against the maximally pessimistic investor beliefs, $\tilde{g}$, I proceed by using $\tilde{g}$ to derive the unique intuitive WPBE outcome.[7]

3.2 Characterizing the Unique Equilibrium

This section builds up to the characterization of the unique intuitive WPBE by demonstrating increasingly more stringent criteria that any intuitive WPBE must satisfy, and then finally demonstrating that only one such equilibrium can satisfy all the necessary criteria.

The analysis begins with a single crossing condition on the financing contract. The interpretation of this condition is that firm's with higher types are more willing to take on additional debt for larger initial loans.

Lemma 3.2. *Fix a firm of type ρ and a number of purchase orders $\hat{\rho}$. Now consider a funding contract (ℓ, d). Next, consider a second funding contract (ℓ', d') such that $d' > d$ and $u_\rho(\hat{\rho}, \ell, d) = u_\rho(\hat{\rho}, \ell', d')$. Then for any firm of type $\rho' > \rho$, $u_{\rho'}(\hat{\rho}, \ell', d') > u_{\rho'}(\hat{\rho}, \ell, d)$.*

Using the single crossing condition, I prove that in any intuitive WPBE different types of firm will never pool by offering the exact same contracts:

[7] Other WPBE exist under less pessimistic beliefs that are still consistent with the intuitive criterion, however these generate the same equilibrium contracts and investor acceptance rule.

Lemma 3.3. *Let* $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}$ *be an equilibrium offered contracts of a firm with* ρ *customers. Then in any intuitive WPBE, for any two different different types* $\rho \neq \rho' \in [0, \overline{\rho}]$*, the two types do not propose identical funding and purchase order contracts:* $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\} \neq \{\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'}\}$*.*

The intuition behind the proof of this result is that if there is a non-singleton group of firms all offering the same set of contracts, then the set of contracts is being approved because the "average type" firm within the group is providing the investor with enough expected revenue that the investor approves the contract. This means that the "above average" types in the group could obtain better financing terms if they could successfully separate from the rest of the group, since they would provide more expected revenue for the investor. I use the single crossing condition to create a contract that the "above average" types in the group strictly prefer and the "below average types" do not prefer which gets approved by the investor.

Notice that this analysis implies that all firms, even those with ρ close to zero, will propose non-zero contracts and operate. This is because there are no fixed costs in the model. With fixed costs there would be some threshold below which all firms would pool by writing no contracts, and this result would instead only be applicable above this operation threshold.

The next condition an intuitive WPBE must satisfy is somewhat less standard. I show that for a fixed purchase order agreement, $\hat{\rho}$, the financing contract has to maximize the firm's expected utility subject to the investor's *full-information* participation condition to be part of an intuitive WPBE. This means that the firm must pick a contract which maximizes their utility subject *only* to the constraint that if the investor knew the firm's type with certainty, the investor would approve the proposed contracts. This is different from the maximization problem which is part of the solution concept definition, where the requirement was that the firm picks contracts that maximize its utility given that the investor approves the contract in equilibrium.

Lemma 3.4. *If* $\{(\hat{\rho}_\rho, \ell_\rho, d_\rho)_{\rho \in P}, a, \tilde{g}\}$ *is an intuitve WPBE then for fixed* ρ *and* $\hat{\rho}_\rho$*, whenever*

$\hat{\rho}_\rho < \rho$, *the funding contract* (ℓ_ρ, d_ρ) *solves*

$$(\ell_\rho, d_\rho) = argmax_{\ell,d\geq 0} u_\rho(\hat{\rho}, \ell, d) \text{ s.t. } u_I(\ell, d, e(\rho, \hat{\rho}, \ell, d)) \geq 0. \tag{8}$$

The purchase order agreement helps alleviate the adverse selection problem faced by the investor by serving as a signal of the firm's type. What this lemma implies is that the purchase orders are the objects generating full separation of types. Additionally, because it is easy to show[8] that in any intuitive WPBE, the participation constraint for the investor is going to bind (i.e. $\int_0^{\overline{\rho}} u_I(\rho, \hat{\rho}, \ell, d)\tilde{h}(\rho|\hat{\rho}, \ell, d)\partial\rho = 0$), this Lemma implies that the purchase orders agreements a firm writes are going to uniquely pin down the firm's financing contract.

Since all of the separation of types is maintained through the purchase order agreements, the next step is to determine what must be true of the purchase order agreements to ensure that firms won't mimic other firms. This is usually maintained by a standard incentive compatibility constraint which requires that each type of firm ρ weakly prefer the contracts they offer in equilibrium to the contracts that any other type ρ' offers. Purchase orders actually weaken this constraint slightly because of the requirement that firms cannot write purchase orders for larger quantities than their customers are interested in (i.e. $\hat{\rho}_\rho \leq \rho$). That means that a firm of type ρ only has to prefer their own contract to the contracts of types ρ' who write $\hat{\rho}_{\rho'} \leq \rho$ purchase order agreements in equilibrium.

When firm's write purchase order agreements $\{\hat{\rho}_\rho\}_{\rho\in[0,\overline{\rho}]}$ and financing contracts following Lemma 3.4, then the following equation ensures that they do not want to deviate slightly downward or upward:

$$\frac{\partial\hat{\rho}}{\partial\rho} = \frac{v}{R} * \frac{k_1 * (v\rho + R\tilde{\rho})^{\frac{2\alpha}{1-2\alpha}}}{1 - k_2 * (v\rho + R\tilde{\rho})^{\frac{2\alpha}{1-2\alpha}}}. \tag{9}$$

where $k_1 = \left[\frac{\alpha\beta}{c^\alpha}\right]^{\frac{1-\alpha}{1-2\alpha}} (1-\alpha)^\alpha$ and $k_2 = k_1 \frac{\alpha}{(1-\alpha)^\alpha}$.

Program 9 is relevant whenever the contract writing strategy, $\{\hat{\rho}_\rho\}_{\rho\in[0,\overline{\rho}]}$ is continuous, for all types ρ who are not writing the maximum number of contracts ($\hat{\rho}_\rho < \rho$). In the cases

[8]See Lemma A.1 in the appendix for details.

where the contract writing strategy must solve Program 9, that strategy will be strictly increasing, and convex. Next note that $\hat{\rho}_{\rho=0} = 0$ because a firm with no customers cannot write any purchase order agreements. I can also show that for all firms in a $\epsilon > 0$ neighborhood around zero, the firm strictly prefers setting $\hat{\rho} = 0$ over setting $\hat{\rho} = \rho$. Hence Equation 1 must hold in the neighborhood around $\rho = 0$, meaning that this uniquely pinned down our candidate equilibrium contracts.

Plugging into Equation 1 the initial point $\hat{\rho}_{\rho=0} = 0$, yields that the derivative of the purchase order writing path is 0 at $\rho = 0$ and goes to infinity as ρ increases. Thus the purchase order writing path will cross the 45^{deg} line for some positive value ρ^*. Then on the range $[0, \rho^*]$ $\hat{\rho}_\rho$ will solve Equation 1 with $\hat{\rho}_0 = 0$. If $\rho^* < \overline{\rho}$ then for all $\rho \in [\rho^*, \overline{\rho}]$ $\hat{\rho} = \rho$. To obtain the associated financing contract each firm performs the maximization in Lemma 3.4 plugging in $\hat{\rho}_\rho$.

Now all that remains to yield the characterization below is to demonstrate that there are no profitable deviations from the candidate equilibrium above under the maximally pessimistic investor beliefs. So let $\tilde{\rho}$ be the solution to Equation 1 with initial condition $\tilde{\rho}_0 = 0$, and $\rho^* > 0$ be the point such that $\tilde{\rho}_{\rho^*} = \rho^*$ Then:

Theorem 3.1. *The unique set of contracts allowable in a WPBE that satisfies the Intuitive Criterion are* $(\hat{\rho}_\rho, \ell_\rho(\hat{\rho}_\rho), d_\rho(\hat{\rho}_\rho))_{\rho \in [0,\overline{\rho}]}$ *where*

$$\hat{\rho}_\rho = \begin{cases} \tilde{\rho}(\rho) & \text{if } \rho \leq \rho^* \\ \rho & \text{if } \rho > \rho^*. \end{cases} \tag{10}$$

$$\ell_\rho = \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha}(v\rho + R\hat{\rho}_\rho) \right]^{\frac{1}{1-2\alpha}}, \tag{11}$$

$$d_\rho = \alpha(v\rho + R\hat{\rho}_\rho). \tag{12}$$

Notice that the threshold type ρ^* need not always be less than $\overline{\rho}$ which means that in some environments all firms will shade down their customers.

3.3 Discussion

The equilibrium takes on a bifurcated structure where firms below the threshold type will shade down the number of purchase orders they write while firms above the threshold write the maximum number of purchase orders they can. To understand why the equilibrium takes its shape it is helpful to analyze the firms above and below the threshold separately.

The firms below the threshold underreport their demand by setting $\hat{\rho} < \rho$ so they don't incur excessive penalties in the case that they fail. Lower type firms exert less effort for a fixed set of contracts than higher type firms meaning their probability of failure is higher. Purchase order do have the side effect of serving as a way for firms to commit to higher levels of effort in an attempt to avoid higher penalties, but in the case of the firms below the threshold, the magnitude of the penalties is too large to make committing to even higher effort valuable. On the other hand firms above the threshold are less affected by the penalty. These firms want to commit themselves to higher effort by increasing the number of purchase orders they write as much as possible, thus exposing themselves to higher penalties in the case of failure. In this sense purchase orders are helping to alleviate the moral hazard problem the investor has by serving as a wedge increasing the difference in firm payoff between success and failure.

In addition to moral hazard considerations, purchase orders are also serving as a way to address the adverse selection problem for the investor. Lemma 3.3 requires that the equilibrium is fully separating, and purchase orders are helping to create that separation. Above the threshold, the separation is simple, all firms are reporting $\hat{\rho} = \rho$ so no firm can mimic a type higher than it; these firms do not want to mimic lower type firms because the contracts offered to those firms are less desirable. Below the threshold the effect of purchase orders isn't nearly as clear cut. As firms write larger purchase orders, the set of potential types this firm could belong to shrinks in a way that benefits the firm because the investor knows the firm is above a certain type. This is beneficial because the when the investor thinks the firm is of a higher type, the investor is more willing to approve funding proposals from the firm. Below the threshold, this positive effect from writing larger purchase orders is being tempered, once again, by the firm's desire to avoid large penalties, but purchase orders are still responsible for much of the separation.

Next I will apply this discussion of how purchase orders affect effort and generate a separation of types to explain why firms in our setting prefer having access to purchase orders over simply having access to traditional debt financing.

4 Purchase Order Financing and Traditional Debt Financing

Now consider the firm's problem without access to purchase order financing. This debt setting becomes a benchmark against which I compare purchase order financing because it is an extremely common form of firm financing, and structurally reasonably similar to purchase order financing. Moreover many of the firms that turn to purchase order financing do so because they cannot obtain adequate loans under traditional debt financing.[9] The comparisons in this section demonstrate how firms with no collateral prefer the loans they get from purchase order finacing to those found under traditional debt financing. First I characterize the optimal separating WPBE in this debt-only environment. From there I compare the debt only equilibrium to the purchase order financing equilibrium characterized in Section 3.

4.1 The Debt-Only Equilibrium

The model for the debt-only environment is the same as the purchase order environment with a restriction that the number of purchase orders written, $\hat{\rho}$, must equal zero for the

[9]See Vermoesen, Deloof, and Lavern (2013) for a discussion of small and medium enterprises loosing access to credit in the aftermath of the 2008 financial crisis and Reindorp et. al. for a discussion of how firms who do not have good access to traditional debt financing might utilitize purchase order financing instead. Martin (2010) describes how Wells Fargo started substituting purchase order loans for traditional loans in 2008 as their credit requirements tightened.

firm regardless of type. As a result, the investor's posterior will now only be affected by the firm's funding proposal since all firms necessarily write no purchase orders.

First I solve for optimal effort. Here the firm maximizes their debt-only utility from Equation 2 over effort choices which yields effort:

$$e_\rho(\ell, d) = [(v\rho - d)\alpha\beta\ell^\alpha/c]^{1/1-\alpha}. \tag{13}$$

Rewriting the utility of the firm of type ρ and the investor gives:

$$u_\rho(\ell, d) = (v\rho - d)f(e_\rho(\ell, d), \ell) - ce_\rho(\ell, d). \tag{14}$$

$$u_\ell(\rho, \ell, d) = (\max\{d, v\rho\})f(e_\rho(\ell, d), \ell) - \ell. \tag{15}$$

From here I solve for equilibrium contracts.

It is helpful to note that Lemma 3.2, the single crossing condition, still holds in this setting when one fixes $\hat{\rho} = 0$ since purchase orders no longer exist. With single crossing still in place I show that in any fully separating WPBE (meaning one where no two types of firm $\rho \neq \rho'$ offer the same (ℓ, d) pair) the loan size ℓ and debt size d are strictly increasing as the firm's level of demand increases.

Lemma 4.1. *Consider a fully separating Debt-Only WPBE,* $\{\{\ell_\rho, d_\rho\}_{\rho\in[0,\bar{\rho}]}, h, a\}$. *Both* ℓ_ρ *and* d_ρ *are strictly increasing in the firm's type,* ρ.

From this point, I derive the unique separating WPBE where the IR constraint for the investor binds. I focus on this equilibrium because it is the optimal fully separating equilibrium for the firm in the Debt-Only environment and because as I show later in this section, there is no intuitive WPBE, so this is the best basis for comparison to the equilibrium derived in Theorem 3.1. Formally:

Proposition 4.1. *There is a unique fully separating WPBE equilibrium in which the investor's IR constraint binds when facing each type* ρ, *meaning Equation 15 equals to zero for* *every value of $\rho \in [0, \bar{\rho}]$. Moreover each type of firm, ρ, prefers there equilibrium contracts in the IR binding equilibrium to their equilibrium contracts in any other fully separating WPBE in the debt-only environment.*

Exploiting the IR constraint pins down ℓ in terms of d and then using the IC constraint on different types for the firm I find that the unique fully separating equilibrium where IR binds takes the following form:

Theorem 4.1. *In the optimal fully separating WPBE for the firm, the IR constraint binds; d_ρ solves the following differential equation*

$$\frac{\partial d}{\partial \rho} = \frac{d * v\alpha^2}{(1-\alpha)[d - \alpha * v\rho],} \tag{16}$$

with $d(0) = 0$. The corresponding funding level is:

$$\ell = \left[\frac{\alpha\beta}{c^\alpha} d(v\rho - d)^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{1-2\alpha}}. \tag{17}$$

Notice that this is not the unique separating WPBE. However, the purpose of this analysis is to show that the firms in this setting prefer Purchase Order Financing to the traditional Debt Financing that they would have available. Since any type of firm strictly prefers the equilibrium where the investor IR constraint binds to any other equilibrium, characterizing this equilibrium suffices for comparison. The previous section did not consider equilibria where the IR constraint was not binding using the intuitive criterion. The issue here is that there are no WPBE that satisfy the intuitive criterion, so there is no longer a guarantee that the IR constraint bind.

Lemma 4.2. *In the Debt-Only setting there exists no Weak Perfect Bayesian Equilibrium that satisfies the intuitive criterion.*

To show why this is true, first note that Lemmas 3.1-3.4 are still applicable with $\hat{\rho}_\rho = 0$

for all types ρ which mean they can be applied here.[10] from there it suffices to show that the intuitive criterion implies the IR constraint must bind and then showing that the unique candidate equilibrium is the one described in Theorem 4.1. The final step is to show that the (ℓ_ρ, d_ρ) pairs laid out in Theorem 4.1 do not satisfy Lemma 3.4 for every type ρ, meaning that they are not always the solution to the firm's maximization problem subject only to the constraint that the firm would approve a contract given that they knew for sure the type requesting it.

4.2 Firm Preference for Purchase Order Financing

Next I consider how firms in this setting rank Purchase Order Financing and Debt Financing, and find that firms of all types prefer Purchase Order Financing.

Proposition 4.2. *Let $E^{PO} = \{(\hat{\rho}_\rho^{PO}, \ell_\rho^{PO}, d_\rho^{PO})_{\rho \in P}, a^{PO}, \tilde{g}^{PO}\}$ be the unique intuitive WPBE in the setting with purchase orders. Next consider any fully separating WPBE in the debt only setting $E^D = \{(\ell_\rho^D, d_\rho^D)_{\rho \in P}, a^D, h^D\}$. For any type ρ, the firm prefers the contracts from the purchase order equilibrium, to the contracts from the Debt equilibrium, that is $u_\rho(\hat{\rho}_\rho^{PO}, \ell_\rho^{PO}, d_\rho^{PO}) \geq u_\rho(0, \ell_\rho^D, d_\rho^D)$.*

Notice that in the debt only setting purchase orders are set equal to zero in the utility function since the firm cannot access them.

To answer why these firms prefer purchase order financing, first consider that in the debt only environment, the only way for the differing types of firm to separate is for the higher types to take on larger levels of debt in order to acquire larger loans. This causes separation because the lower types would not accept such a large level of debt because the marginal value of additional money in the loan is lower for lower type firms.

Once the firm has access to purchase orders as well, they have an additional method of signalling through the purchase orders. The purchase orders act as signals in two different

[10] Also separately proved in section [INSERT REF] of the appendix

ways. First and more obviously, the purchase order indicates a minimum level of demand for the firm's product; the firm must have a demand of at least $\hat{\rho}$. Second, since firms of higher types exert more effort for fixed purchase order and financing contracts, the penalty, R, embeded in each purchase order is less detrimental to higher type firms. This means that as firm type increases, firms write larger and larger purchase orders, even when they aren't constrained by the restriction that $\hat{\rho} \leq \rho$.

When the firm can use purchase orders to create separation of types within the equilibrium, they are free to set the loan size and debt amount equal to more favorable values. Recall Lemma 3.4 said that in the intuitive WPBE with Purchase Orders, the firm will write their purchase orders and then choose the maximal (ℓ, d) contract that the investor would approve were they to have full information regarding the firm's type. In essence the purchase order is giving the firm an extra degree of freedom to completely deal with the adverse selection problem, and from there the firm is free to optimize as if there were no other types they could be confused with. This is why firms prefer their contracts when purchase orders are permitted.

5 Endogenizing the Penalty in Purchase Order Agreements

Until this point, this paper has assumed that the penalty, R, that the firm faces when it fails to deliver on a purchase order is reputational or associated with legal costs. On the other hand one might consider the case where R is a monetary fine paid to the customers. In that case, the exact size of the fine, R, would be selected as one of the terms in the Purchase Order Agreement.

When the firm is given two dimensions along which to write the purchase order agreements, both the quantity, $\hat{\rho}$, and the penalty, R, they can separately calibrate $\hat{\rho}$ as a signal of demand, and R as a mechanism for committing themselves to higher levels of effort. Since the set of contracts a firm can get approved for is expanding as the firm's type goes up, it is always better for the firm if the investor believes the firm to have as high a type as possible. Thus the firm should always write $\hat{\rho}_\rho = \rho$ purchase order agreements.

If the firm writes $\hat{\rho}_\rho = \rho$ purchase order agreements then the investor has a perfect signal of the firm's type and there is no longer any adverse selection at play. This problem reduces to a simple three stage maximization problem where by backwards induction, first the firm solves for the optimal effort given a set of financing contracts and purchase order agreements. Next the firm picks the optimal financing contract subject to the investor's participation constraint, plugging in the effort from the previous step, in terms of purchase order agreements. Finally, the firm selects the optimal penalty, R. This yields a simple closed-form solution:

Proposition 5.1. *In the Endogenous Penalty setting the unique WPBE that satisfies the intuitive criterion is* $(R_\rho, \hat{\rho}_\rho, \ell_\rho(R_\rho, \hat{\rho}_\rho), d_\rho(R_\rho, \hat{\rho}_\rho))_{\rho\in[0,\overline{\rho}]}$ *where*

$$\hat{\rho}_\rho = \rho,$$

$$R_\rho = \max\left\{\left[\frac{(1-2\alpha)c^{\frac{\alpha}{1-\alpha}}}{[(\alpha^\alpha-\alpha)\beta]^{\frac{1}{1-\alpha}}}\right]^{\frac{1-2\alpha}{2\alpha}}\left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha}\right]^{\frac{1}{2\alpha}}\rho^{\frac{1-2\alpha}{2\alpha}} - v, 0\right\},$$

$$\ell_\rho = \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha}(v\rho + R_\rho\rho)\right]^{\frac{1}{1-2\alpha}},$$

$$d_\rho = \alpha(v\rho + R_\rho\rho).$$

Note that the penalty, R, is increasing in the firm's type ρ, because as the firm's type increases, the firm's effort for a fixed set of contracts also increases. This means that when the firm trades off risk of penalty versus the advantages from committing themselves to even higher effort, the risk of penalty is going to matter less than the additional benefits of effort. Moreover the additional return to effort of increasing the penalty is higher for firms of higher type since R is multiplied by ρ in the effort equation. Section 3 highlighted the importance of the penalty in addressing the moral hazard present in purchase order financing. This Proposition highlights how purchase order financing helps alleviate moral hazard alongside adverse selection.

6 Discussion and Conclusion

6.1 Differences Between Purchase Order Financing and Other Sources of Alternative Funding

Here I discuss three non-traditional types of funding similar to purchase order financing, crowd-funding, buyer direct financing, and factoring. I highlight the difference between these funding sources and purchase order financing.

Crowd-Funding is a well-known alternative source of business funding which is reasonably similar to purchase order financing. Like purchase order financing, it leverages the presence of interested customers to acquire the money necessary to produce a good. Additionally, as consumer demand increases, the amount of money the firm can access will increase. However, there are several differences between Purchase Order Financing and Crowd-Funding Verntures (for example the websites Kickstarter and Indiegogo). First in the case of crowd-funding, interested customers pay the firm up-front and in return, the firm pledges to deliver the good if successfully produced.[11] This means that, unlike the case of Purchase Order Financing, once a firm has its interested customers in place, they do not have to look for an outside investor for funding. The second, related difference is that if the firm fails to develop and produce their proposed product, it is the customer who loses their investment, often with little recourse. Finally, crowd-funding usually relies on hundreds or thousands of small investments from individuals who want to buy one product, whereas purchase order financing requires a purchase order from one or two large vendors who the investor has verified the

[11] For a more in-depth exploration of crowd-funding firms, see Strauz (2017) or Deb, Ory, and Williams (2021).

creditworthiness of. This means that crowd-funding produces goods that are sold direct-to-consumer, while purchase order financing is only viable for business to business sales.

Buyer Direct Financing is similar to crowd-funding, but like purchase orders, it is intended for business to business transactions, not direct-to-consumer sales. In Buyer Direct Financing, a customer both commits to purchasing a certain volume of the firm's product, and finances the firm's production. So like with crowd-funding, under Buyer Direct Financing the firm does not need to find a third party investor to fund their production costs. Tang, Yang, and Wu (2018) find that customers prefer purchase order financing to buyer direct financing when the customer is unsure how likely the firm is to deliver on the good. In these cases the buyer wants to 'outsource' that risk to an investor who has more experience calculating how much risk is involved in lending to given firms.

Factoring is a well established form of financing which closely resembles purchase order financing with one major difference. In purchase order financing, the investor secures the loan with a purchase order agreement, and the purpose of the loan is for the firm to obtain funding for production. In Factoring, the investor secures the loan using an invoice the firm sends the customer post production. This means that the firm has already successfully funded this round of production, and usually the purpose of Factoring is to get cash in hand sooner to fund future business the firm has. This means that the only risk investors carry with Factoring is that the customer might refuse to pay. Since Purchase Order Financing also carries the larger risk that they will fail to successfully deliver the goods to the customer, rates for purchase order financing are typically less favorable than those associated with Factoring.

6.2 Conclusion

This paper demonstrates how firms with little collateral turn to purchase order financing over more traditional debt financing because purchase order financing allows the firms to acquire larger loans. This is for two primary reasons; first purchase orders serve as signals of the firms type. Under traditional debt financing the only way that firms can signal their type to investors is via their offers for loan size and debt. This causes the firms to take on higher rates of debt, reducing the level of loan they can shoulder. With purchase orders in place, the firm has a different channel through which they can signal their type, allowing them to take on larger loans for comparable interest rates. This is especially highlighted by the firms with larger customer bases who fully report their customers to the investor giving a perfectly revealing signal of their demand; since no firms of lower type can mimic these firms, they have the freedom to propose a contract as if there were no adverse selection problem for the investor. The second feature of purchase order financing that helps it supply larger loans to firms with little to no collateral, is the way in which purchase order agreements commit the firm to higher levels of effort, thus helping to alleviate the investor's moral hazard concerns. In traditional debt financing there is less of a threat of reputational penalties should the company not succeed in this round, since they haven't guaranteed to a customer the delivery of the good. Since the investor is more afraid the firm will shirk under debt financing, they approve the firm for smaller loans. As purchase order financing does more to alleviate the adverse selection and moral hazard concerns of the investor than traditional debt financing, it opens firms up to larger loans, and is more appealing to firms with no collateral as credit markets tighten.

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A Appendix

A.1 Proofs for Section 3

A.1.1 Proof of Lemma 3.1

Given an intuitive WPBE $E^h = (\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho\in[0,\bar{\rho}]}, a_h, h)$, I show that $\{\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho\in[0,\bar{\rho}]}, \tilde{a}_{\tilde{g}}, \tilde{g}\}$ is also an intuitive WPBE.

Both h and $\tilde{g}$ are derived using Bayes rule along the path of play so they are identical along the path of play. Hence the acceptance rule is identical along the path of play.

Now I consider off-path actions and show that they cannot be profitable deviations. Note that having an offer rejected will not be a profitable deviation because this is weakly inferior to the firm offering $(0, 0, 0)$ which will always be accepted. Thus all profitable deviations must be offers that are accepted by the investor. Then consider a potential profitable deviation, I want to show that it will not be accepted under $\tilde{g}$ and $a_{\tilde{g}}$. If $(\hat{\rho}', \ell', d')$ is a candidate profitable deviation from $\{\hat{\rho}_\rho, \ell_\rho, d_\rho\}_{\rho\in[0,\bar{\rho}]}$, we know that it was rejected by investors under a_h. The acceptance rule requires the investor to reject if and only if

$$\int_{\hat{\rho}'}^{\bar{\rho}} [d'\beta\ell'^\alpha e_\rho(\hat{\rho}', \ell', d')^\alpha - \ell']\, h(\rho)d\rho = d'\beta\ell'^\alpha \int_{\hat{\rho}}^{\bar{\rho}} e_\rho(\hat{\rho}', \ell', d)^\alpha h(\rho)d\rho - \ell < 0$$

Similarly under acceptance rule $a_{\tilde{g}}$, the investor rejects iff

$$\int_{\hat{\rho}'}^{\bar{\rho}} \left[d'\beta\ell'^{\alpha}e_{\rho}(\hat{\rho}',\ell',d')^{\alpha} - \ell'\right] h(\rho)d\rho = d'\beta\ell'^{\alpha}\int_{\hat{\rho}}^{\bar{\rho}} e_{\rho}(\hat{\rho}',\ell',d)^{\alpha}\tilde{g}(\rho)d\rho - \ell < 0$$

So to show that here $a_{\tilde{g}}$ requires rejection whenever a_h does it suffices to show that

$$\int_{\hat{\rho}}^{\bar{\rho}} e_{\rho}(\hat{\rho}',\ell',d)^{\alpha}h(\rho)d\rho \geq \int_{\hat{\rho}}^{\bar{\rho}} e_{\rho}(\hat{\rho}',\ell',d)^{\alpha}\tilde{g}(\rho)d\rho \tag{18}$$

Recall that for E^h to be an intuitive WPBE, it must be that for any off path $(\hat{\rho}',\ell',d')$, $supp(h(\rho|\hat{\rho}',\ell',d')) \subset P = \{\rho|u_{\rho}(\hat{\rho},\ell,d) \geq u_{\rho}(\hat{\rho}_{\rho},\ell_{\rho},d_{\rho})\} \cap \{\rho \geq \hat{\rho}\}$ when P is non empty and $supp(h(\rho|\hat{\rho}',\ell',d')) \subset [\hat{\rho}',\bar{\rho}]$ when P is empty. But by Definition 3.2, whenever P is non-empty, $\tilde{g}(\rho|\hat{\rho}',\ell',d') = \delta_{inf(P)}$ and whenever P is empty, $\tilde{g}(\rho|\hat{\rho}',\ell',d') = \delta_{\hat{\rho}'}$. Hence $\tilde{g}(\rho|\hat{\rho}',\ell',d') = \delta_{inf(supp(h(\rho|\hat{\rho}',\ell',d')))}$. Finally recall that $e_{\rho}(\hat{\rho}',\ell',d')$ is strictly increasing in ρ. Hence Equation 18 holds so $E^{\tilde{g}} = (\{\hat{\rho}_{\rho},\ell_{\rho},d_{\rho}\}_{\rho\in[0,\bar{\rho}]}, a_{\tilde{g}}, \tilde{g})$ is also an intuitive WPBE.

A.1.2 Proof of Lemma 3.2

Plugging in the optimal effort choice of the start-up into their payoff function yields

$$U_{\rho}(\ell,d) = \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right]\left(\frac{\beta}{c^{\alpha}}\right)^{\frac{1}{1-\alpha}} \ell^{\frac{\alpha}{1-\alpha}} (v\rho + R\hat{\rho} - d)^{\frac{1}{1-\alpha}} - R\hat{\rho}$$

Now we fix ρ and consider a (ℓ,d) and (ℓ',d') such that $d' > d$ and $U_{\rho}(I,d) = U_{\rho}(\ell',d')$. Notice that by the monotonicity of U, $\ell' > \ell$. In fact d' uniquely pins down ℓ'. Next consider an arbitrary $\rho' > \rho$.

We want to show that $U_{\rho'}(\ell,d,) < U_{\rho'}(\ell',d')$. Note this will be true so long as

$$(\ell')^{\frac{\alpha}{1-\alpha}} (v\rho' + R\hat{\rho} - d')^{\frac{1}{1-\alpha}} > \ell^{\frac{\alpha}{1-\alpha}} \left(v\rho' + R\rho\hat{-}d\right)^{\frac{1}{1-\alpha}}$$

Because $U_\rho(\ell, d,) = U_\rho(\ell', d')$

$$(\ell')^{\frac{\alpha}{1-\alpha}} (v\rho + R\hat{\rho} - d')^{\frac{1}{1-\alpha}} = \ell^{\frac{\alpha}{1-\alpha}} (v\rho + R\hat{\rho} - d)^{\frac{1}{1-\alpha}}$$

Rewriting in terms of ℓ' yields

$$(I')^{\frac{\alpha}{1-\alpha}} = I^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho + R\hat{\rho} - d}{v\rho + R\hat{\rho} - d'}\right]^{\frac{1}{1-\alpha}}$$

Then

$$(I')^{\frac{\alpha}{1-\alpha}} (v\rho' + R\hat{\rho} - d')^{\frac{1}{1-\alpha}} = I^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho + R\hat{\rho} - d}{v\rho + R\hat{\rho} - d'}\right]^{\frac{1}{1-\alpha}} (v\rho' + R\hat{\rho} - d')^{\frac{1}{1-\alpha}}$$

Notice that because $(v\rho') > (v\rho')$, $d' > d$, and $\frac{1}{1-\alpha} > 0$:

$$\left[\frac{v\rho + R\hat{\rho} - d}{v\rho + R\hat{\rho} - d'}\right]^{\frac{1}{1-\alpha}} > \left[\frac{v\rho' + R\hat{\rho} - d}{v\rho' + R\hat{\rho} - d'}\right]^{\frac{1}{1-\alpha}}$$

Hence

$$\ell^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho + R\hat{\rho} - d}{v\rho + R\hat{\rho} - d'}\right]^{\frac{1}{1-\alpha}} (v\rho' + R\hat{\rho} - d')^{\frac{1}{1-\alpha}} > \ell^{\frac{\alpha}{1-\alpha}} (v\rho' + R\hat{\rho} - d)^{\frac{1}{1-\alpha}}$$

This yields the final result.

A.1.3 Proof of Lemma 3.3

Suppose by contradiction that there are types $\rho \neq \rho'$ where the financing contracts the start-ups propose are identical.

We know that the effort that the start-up is willing to exert is $e = [(v\rho - d - R\hat{\rho})\alpha\beta\ell^\alpha/c]^{1/1-\alpha}$. Note that this is increasing in ρ for fixed financing contracts $\{\ell, d\}$. Call the set of all types propising the same contract $R \subset [0, \overline{\rho}]$. Note that ρ, ρ' all propose financing contract $\{\ell, d\}$. Then it must be that the expected payoff to the venture capitalist given the distribution of types in R, $G(\rho)$ is weakly positive, meaning:

$$\int_{\rho\in R} d\beta\ell^{\alpha}[(v\rho - d)\alpha\beta\ell^{\alpha}/c]^{\alpha/1-\alpha}dG(\rho) \geq \ell$$

This was derived by plugging in the effort level a start-up of type ρ is willing to exert under contracts $\{\ell, d\}$, $e = [(v\rho - d - R\hat{\rho})\alpha\beta\ell^{\alpha}/c]^{1/1-\alpha}$ into the VC's payoff function. Note that this is increasing in ρ for fixed financing contracts $\{\ell, d\}$.

Rearranging we get:

$$d\beta\left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}}\int_{\rho\in R}(v\rho - d)^{\alpha/1-\alpha}dG(\rho) = \ell^{\frac{1-2\alpha}{1-\alpha}}$$

The strategy moving forward is to create a contract that a type 'above the mean' of R would take but no one below would. Now consider the types $\rho' \in R$ such that

$$(v\rho' - d)^{\alpha/1-\alpha} > \int_{\rho\in R}(v\rho - d)^{\alpha/1-\alpha}dG(\rho).$$

There are two cases: the case where there exists such a $\rho' \neq sup_{\rho}\{R\}$ and the case where the only such $\rho' = sup_{\rho}\{R\}$. Now consider the first case. Let u be the utility that the start-up of type ρ' gets from offering the pooling contracts $\{\hat{\rho}_R, \ell_R, d_R\}$. However these contracts aren't the only contracts that yield utility u. Consider the set of all financing contracts $\{\ell', d'\}$ that yield u for type ρ'. This set of contracts is all $\{\ell, d\}$ that satisfy:

$$U_{\rho'}(\ell', d') = \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right]\left(\frac{\beta}{c^{\alpha}}\right)^{\frac{1}{1-\alpha}}\ell'^{\frac{\alpha}{1-\alpha}}(v\rho' - d')^{\frac{1}{1-\alpha}} = u$$

Now we pick an $\{\ell'_{\epsilon}, d'_{\epsilon}\}$ from this set such that $\ell'_{\epsilon} = \ell_R + \epsilon$ for some $\epsilon > 0$ and d' is the corresponding debt level that allows $\{\ell', d'\}$ to satisfy the above equation. Note that by the structure of the utility function, to maintain the same utility level, if ℓ increases, d must also increase. This means that $d'_{\epsilon} > d_R$, and d'_{ϵ} is increasing in ϵ. Then using our single crossing condition we find that for any $\rho < \rho'$, the start-up will strictly prefer the original funding contract $\{\ell_R, d_R\}$ to $\{\ell'_\epsilon, d'_\epsilon\}$. Then by the intuitive criterion if the venture capitalist sees $\hat{\rho}_R$ and is proposed funding contract $\{\ell', d'\}$, by the intuitive criterion, they must believe that $\rho \geq \rho'$.

The 'worst case' beliefs, $\tilde{g}$ for the venture capitalist put probability 1 on the start-up being of type ρ'. Then we claim that for sufficiently small $\epsilon > 0$, the venture capitalist's expected payoff from approving funding contract $\{\ell'_\epsilon, d'_\epsilon\}$ is greater than zero.

To show this, simply recall that the payoff to the venture capitalist from the original $\{\ell_R, d_R\}$ was strictly positive when the start-up was of type ρ'.

$$d_R\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho' - d_R)^{\frac{\alpha}{1-\alpha}} - \ell_R^{\frac{1-2\alpha}{1-\alpha}} = \eta > 0$$

Then by the continuity of the start-up's and venture capitalist's payoffs in ℓ and d, there must be an $\epsilon > 0$ sufficiently small that

$$d'_\epsilon\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho - d'_\epsilon)^{\frac{\alpha}{1-\alpha}} - \ell'_\epsilon{}^{\frac{1-2\alpha}{1-\alpha}} > 0$$

So under beliefs $\tilde{g}$, the venture capitalist will accept the proposal. Hence all start-ups in R with type $\rho > \rho'$ have a profitable deviation.

For case 2 where there was only one 'above the mean' type, note that this means $sup_R\rho = \sup\{R\} \in R$. Additionally note that $sup\{R\} > \sup\{R \backslash sup\{R\}\} = \tilde{\rho}$. Then we pick some $\rho' \in (\tilde{\rho}, sup\{R\})$ (clearly $\rho' \notin R$ but the single crossing condition will still hold.) Repeat the analysis noting that a start-up of type $sup\{R\} \in R$ would have the profitable deviation and that $\tilde{g}$ implies the venture capitalist must believe for certain that the start-up is at least of type ρ'.

Thus R must be a singleton, meaning there is no pooling.

A.1.4 Proof of Lemma 3.4

By contradiction - Assume that for some $\rho \in [0, \overline{\rho}]$, we have that (ℓ_ρ, d_ρ) doesn't maximize the start-up's expected utility over contracts that the venture capitalist would approve just for start-up's of type ρ. Mathematically:

$$(\ell_\rho, d_\rho) \neq \text{argmax}_{\ell,d}(v\rho - d)f(e_\rho(\ell, d), \ell) - ce_\rho(\ell, d)$$
$$s.t. \quad d * f(e_\rho(\ell, d), \ell) - \ell \geq 0. \quad (19)$$

Then let (ℓ', d') be the financing contract that does solve Equation **??**. Starting from (ℓ', d'), increase d' to d''_ϵ such that $u_\rho(\ell_\rho, d_\rho) + \epsilon = u_\rho(\ell', d''_\epsilon)$ for some small $\epsilon > 0$. Now pick a small $\delta(\epsilon) > 0$ and consider type $\tilde{\rho} = \rho - \delta(\epsilon)$. We increase both d''_ϵ by some $\eta > 0$ to $\tilde{d}$ and then pick a corresponding $\tilde{\ell}$ which makes the start-up of type $\tilde{\rho}$ exactly indifferent between the contract meant for type ρ, and $\tilde{\ell}, \tilde{d}$, so $u_{\tilde{\rho}}(\ell_\rho, d_\rho) = u_{\tilde{\rho}}(\tilde{\ell}, \tilde{d})$. Notice that for a fixed d''_ϵ, $\tilde{\ell}(\eta)$ will be strictly increasing in η, and as $\eta \to 0$, $\tilde{\ell}(\eta) \to \ell'$.

Because $\tilde{d} > d''_\epsilon$ and $u_{\tilde{\rho}}(\ell_\rho, d_\rho) = u_{\tilde{\rho}}(\tilde{\ell}, \tilde{d})$, by the single crossing property of the start-up's utility, any start-up with fewer than $\tilde{\rho}$ customers will strictly prefer the contract originally prescribed to type ρ to the new contract $(\tilde{\ell}, \tilde{d})$, and any start-up with more than $\tilde{\rho}$ customers will strictly prefer the new contract.

Since this is an equilibrium, all types below $\tilde{\rho}$ prefer their own equilibrium contracts to the contracts specified for ρ. Hence if the venture capitalist sees someone propose the contract $(\tilde{I}, \tilde{d})$, their belief, $\tilde{g}$, is that the start-up cannot be of type $\rho' < \tilde{\rho}$. Under the maximally pessimistic beliefs admittable under the intuitive criterion, $\tilde{g}$, the venture capitalist believes they are facing a start-up of type $\tilde{\rho}$ with probability 1.

Finally, since (ℓ', d') satisfied equation **??**, the venture capitalist had non-negative expected profit from (ℓ', d'). By increasing d' to d''_ϵ while holding I' fixed, the contract (ℓ', d''_ϵ) gave the venture capitalist strictly positive profit for type ρ. Then because utility is continuous in ℓ, d, and ρ for a sufficiently small ϵ, $\delta(\epsilon)$, and η, the contract $(\tilde{\ell}, \tilde{d})$ also yields positive profits for the venture capitalist if they believe the start-up is of type $\tilde{\rho}$ with probability 1.

Hence, if the Venture Capitalist sees the contract $(\tilde{I}, \tilde{d})$, they will approve it under belief $\tilde{g}$. This means the start-up of type ρ has a profitable deviation to $(\tilde{I}, \tilde{d})$. **Contradiction.**

A.1.5 Proof of Theorem 3.1

Lemma A.1. *In any WPBE that satisfies the intuitive criterion the IR constraint for the venture capitalist must bind.*

Proof - First note from the previous lemma that any WPBE that satisfies the intuitive criterion, then every type ρ offers a different contract $\{\ell_\rho, d_\rho\}$. So if the venture capitalist see a contract $\{\ell_\rho, d_\rho\}$, they know for certain that the start-up is of type ρ. Suppose by contradiction that for some type ρ, the contract that the start-up offers $\{\ell_\rho, d_\rho\}$ such that the IR constraint on the VC doesn't bind, that is:

$$d_\rho \beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho + R\hat{\rho} - d_\rho)^{\frac{\alpha}{1-\alpha}} - \ell_\rho^{\frac{1-2\alpha}{1-\alpha}} > 0. \tag{20}$$

First notice that fixing $\{\ell_\rho, d_\rho\}$ the venture capitalist's expected payoff is strictly increasing in ρ. Next we pick a $\tilde{\rho}(\epsilon) = \rho - \epsilon$ where ϵ sufficiently small that the contract $\{\ell_\rho, d_\rho\}$ also gives the venture capitalist positive expected surplus when faced with type $\tilde{\rho}_\epsilon$. Now set $d'_\eta = d_\rho + \eta$ and increase ℓ_ρ to ℓ'_η such that $u_{\tilde{\rho}_\epsilon}(\ell'_\eta, d'_\eta) = u_{\tilde{\rho}_\epsilon}(\ell_\rho, d_\rho)$.

Since $\{\ell_\rho, d_\rho\}$ under type $\tilde{\rho}_\epsilon$ gave the VC strictly postive surplus, by the continuity of the venture capitalist's utility, the contract $\{\ell'_\eta, d'_\eta\}$ will also give the VC weakly positive surplus for sufficiently small η. By the single crossing condition all types $\rho' < \tilde{\rho}_\epsilon$ strictly prefer $\{\ell_\rho, d_\rho\}$ to $\{\ell'_\eta, d'_\eta\}$. Moreover because this is a fully separating equilibrium, all such start-ups ρ' weakly prefer their equilibrium contract to $\{\ell_\rho, d_\rho\}$. Thus the 'worst case' beliefs $\tilde{g}$ imply that the venture capitalist believes the start-up must be of at least type $\tilde{\rho}_\epsilon$. This means that the VC will approve a contract $\{\ell'_\eta, d'_\eta\}$ because he expects to get weakly positive surplus.

Again by the single crossing condition we know that type ρ strictly prefers $\{\ell'_\eta, d'_\eta\}$ to $\{\ell_\rho, d_\rho\}$. Since type ρ strictly prefers $\{\ell'_\eta, d'_\eta\}$ (a contract that gets approved) to his equilibrium contract $\{\ell_\rho, d_\rho\}$, ρ has a profitable deviation. **Contradiction.**

Next we solve for the unique candidate equilibrium that satisfies the properties from Lemmas 3.2-3.4 and A.1.

Plugging in the formulas for $d_{\rho'}$ and $\ell_{\rho'}$ from Equations 11 and **??** to the firm's utility function along with $\hat{\rho}_{\rho'}$. Taking the derivative with respect to ρ', set it equal to zero and then plugging in $\rho' = \rho$ yields Equation 22. This constitutes the candidate equilibrium.

Finally we show that there are no profitable deviations in this candidate equilibrium, hence it is an intuitive WPBE equilibrium. Because it was the unique candidate, it is the unique intuitive equilibrium.

Suppose by contradiction there exists a profitable deviation for some type ρ, which we will call $(\hat{\rho}', \ell', d')$. This deviation will fall onto one of the following cases:

1. $\hat{\rho}' = \hat{\rho}_\rho$

2. $\hat{\rho}' = \hat{\rho}_{\rho'}$ for some $\rho' > \rho$.

3. $\hat{\rho}' = \hat{\rho}_{\rho'}$ for some $\rho' < \rho$.

Case 1 - $u_\rho(\hat{\rho}_\rho, \ell_\rho, d_\ell)$ By Lemma 3.4, fixing $\hat{\rho}$, ℓ_ρ, d_ρ maximizes $u_\rho(\hat{\rho}_\rho, \ell, d)$ for any set of contracts that the investor approves. Thus no profitable deviation can exist under $\hat{\rho}_\rho$.

Case 2 - $\hat{\rho}' = \hat{\rho}_{\rho'}$ for some $\rho' > \rho$. Then we know that $u_\rho(\hat{\rho}_{\rho'}, \ell', d') > u_\rho(\hat{\rho}_{\rho'}, \ell_\rho, d_\rho)$. We know that in our candidate equilibrium no type wants the in equilibrium contract of another type by Equation [CODE REF]. Thus $u_\rho(\hat{\rho}_{\rho'}, \ell', d') > u_\rho(\hat{\rho}_\rho, \ell_\rho, d_\rho) \geq u_\rho(\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'})$.

Indifference curves for ρ take the following form:

$$\ell^{\frac{\alpha}{1-\alpha}} = \frac{U + R\hat{\rho}}{k(v\rho + R\hat{\rho} - d)^{\frac{1}{1-\alpha}}} \tag{21}$$

Let $\bar{U}_{\rho'} = u_{\rho'}(\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'})$ and let $\bar{U}_{\rho} = u_{\rho}(\hat{\rho}_{\rho'}, \ell_{\rho'}, d_{\rho'})$. Similarly let $U'_{\rho'} = u_{\rho'}(\hat{\rho}_{\rho'}, \ell', d')$ and let $U'_{\rho} = u_{\rho}(\hat{\rho}_{\rho'}, \ell', d')$. Then we know that

$$\ell_{\rho'}^{\frac{\alpha}{1-\alpha}} = \frac{\bar{U}_{\rho'} + R\hat{\rho}_{\rho'}}{k(v\rho' + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{1}{1-\alpha}}} = \frac{\bar{U}_{\rho} + R\hat{\rho}_{\rho'}}{k(v\rho + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{1}{1-\alpha}}} < \frac{U'_{\rho} + R\hat{\rho}_{\rho'}}{k(v\rho + R\hat{\rho}_{\rho'} - d')^{\frac{1}{1-\alpha}}} \tag{22}$$

The IR condition has to bind for the equilibrium contracts from ρ', meaning

$$d_{\rho'}\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho' + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{\alpha}{1-\alpha}} - \ell_{\rho'}^{\frac{1-2\alpha}{1-\alpha}} = 0.$$

Rewrite as

$$d_{\rho'}\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho' + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{\alpha}{1-\alpha}} - \left[\frac{\bar{U}_{\rho'} + R\hat{\rho}_{\rho'}}{k(v\rho' + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{1}{1-\alpha}}}\right]^{\frac{1-2\alpha}{\alpha}} = 0.$$

Now suppose that the IR constraint is satisfied for $(\hat{\rho}_{\rho'}, \ell', d')$ when the investor thinks the firm is of type ρ.

$$d'\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho + R\hat{\rho}_{\rho'} - d')^{\frac{\alpha}{1-\alpha}} - \left[\frac{U'_{\rho} + R\hat{\rho}_{\rho'}}{k(v\rho + R\hat{\rho}_{\rho'} - d')^{\frac{1}{1-\alpha}}}\right]^{\frac{1-2\alpha}{\alpha}} = 0.$$

Combining yields:

$$d'(v\rho + R\hat{\rho}_{\rho'} - d')^{\frac{\alpha}{1-\alpha}} > d_{\rho'}(v\rho' + R\hat{\rho}_{\rho'} - d_{\rho'})^{\frac{\alpha}{1-\alpha}}$$

By Equation 22 this means that $d' > d'_{\rho}$. However this violates single crossing. **Contradiction**.

Case 3 - The argument is analogous to the argument for case 2 with signs reversed.

A.2 Proofs from Section 4

A.2.1 Proof of Lemma 4.1

Consider a fully separating WPBE $\{\{\ell_\rho, d_\rho\}_{\rho\in[0,\bar{\rho}}, h, a\}$. Suppose by contradiction that $\exists$ $\rho' > \rho$ such that $\ell_\rho \geq \ell_{\rho'}$. There are two cases here:

Case 1 - $d_\rho \geq d_{\rho'}$ - Since this is a WPBE we know that type ρ must weakly prefer their own equilibrium contracts to the equilibrium contracts for type ρ'. Since this is a fully separating equilibrium we know that we cannot have $d_\rho = d_{\rho'}$ and $\ell_\rho = \ell_{\rho'}$ so we know that either $d_\rho > d_{\rho'}$ or $\ell_\rho > \ell_{\rho'}$. If $d_\rho > d_{\rho'}$ but $\ell_\rho = \ell_{\rho'}$ then because U_ρ is strictly increasing in ℓ and strictly decreasing in d for any type ρ, both types will strictly prefer the contracts meant for type ρ' giving type ρ a profitable deviation. If $d_\rho = d_{\rho'}$ but $\ell_\rho > \ell_{\rho'}$ then by the same logic both types will strictly prefer the contracts meant for type ρ giving type ρ' a profitable deviation.

Finally consider the case where $d_\rho > d_{\rho'}$ and $\ell_\rho > \ell_{\rho'}$. Then $U_\rho(\ell_\rho, d_\rho) > U_\rho(\ell_{\rho'}, d_{\rho'})$. By the single crossing property Lemma 3.2 fixing $\hat{\rho} = 0$, we know that since $\rho' > \rho$ and since $\ell_\rho > \ell_{\rho'}$ and $d_\rho > d_{\rho'}$ then we must have $U_{\rho'}(\ell_\rho, d_\rho) > U_{\rho'}(\ell_{\rho'}, d_{\rho'})$. Thus we have a profitable deviation of type ρ' to the equilibrium contracts meant for type ρ.

Case 2 - $d_\rho < d_{\rho'}$ - Here $\ell_\rho \geq \ell_{\rho'}$ AND $d_\rho < d_{\rho'}$. Since U_ρ is strictly increasing in ℓ and strictly decreasing in d for any type ρ, type ρ' will strictly prefer $\{\ell_\rho, d_\rho\}$ to their own equilibrium contract. Since the investor approves $\{\ell_\rho, d_\rho\}$ when offered because it is type ρ's equilibrium contract, this is a profitable deviation for ρ'.

Hence in both case 1 and case 2, at least one type of firm has a profitable deviation so this cannot be a WPBE **Contradiction**.

A.2.2 Proof of Theorem 4.1

We show that there is a unique fully separating candidate equilibrium where the IR binds. Note that IR binding implies:

$$d\beta\left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}}(v\rho-d)^{\frac{\alpha}{1-\alpha}}-\ell^{\frac{1-2\alpha}{1-\alpha}}=0.$$

The firm's utility function selecting some $d_{\rho'}$ will be

$$U_\rho(\ell,d)=\left[\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\right]\left(\frac{\beta}{c^\alpha}\right)^{\frac{1}{1-\alpha}}\left[d_{\rho'}\beta\left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}}(v\rho'-d_{\rho'})^{\frac{\alpha}{1-\alpha}}\right]^{\frac{\alpha}{1-2\alpha}}(v\rho-d_{\rho'})^{\frac{1}{1-\alpha}} \quad (23)$$

Suppose we have an equilibrium path of $\{d_\rho\}_{\rho\in[0,\bar{\rho}}$. We need a condition to make sure no one deviates. We do this by taking the derivative of the utility with respect to ρ', setting it equal to zero and then plugging in $\rho'=\rho$. The derivative becomes

$$\frac{\partial U_\rho}{\partial\rho'}=k\frac{1}{1-\alpha}(v\rho-d'_\rho)^{\frac{1}{1-\alpha}-1}\left(d_\rho(v\rho'-d_{\rho'})^{\frac{\alpha}{1-\alpha}}\right)+$$
$$k\frac{\alpha}{1-2\alpha}\left(d_{\rho'}(v\rho'-d'_\rho)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\alpha}{1-2\alpha}}\left[\frac{\partial d_{\rho'}}{\partial\rho'}(v\rho'-d_{\rho'})^{\frac{\alpha}{1-\alpha}}+\frac{\alpha}{1-\alpha}(v\rho'-d_{\rho'})^{\frac{\alpha}{1-\alpha}-1}\left(v-\frac{\partial d_{\rho'}}{\partial\rho'}\right)d_{\rho'}\right](v\rho-d'_\rho)^{\frac{1}{1-\alpha}} \quad (24)$$

where k is all constant terms which will cancel out. Setting $\frac{\partial U_\rho}{\partial\rho'}=0$ and plugging in $\rho'=\rho$ yields Equation 16.

A.2.3 Redoing the Section 3 Propositions with no Purchase Orders

Proof of Lemma 3.2 - Plugging in the optimal effort choice of the start-up into their payoff function yields

$$U_\rho(\ell,d)=\left[\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\right]\left(\frac{\beta}{c^\alpha}\right)^{\frac{1}{1-\alpha}}\ell^{\frac{\alpha}{1-\alpha}}(v\rho-d)^{\frac{1}{1-\alpha}}$$

Now we fix ρ and consider a (ℓ, d) and (ℓ', d') such that $d' > d$ and $U_\rho(I, d) = U_\rho(\ell', d')$. Notice that by the monotonicity of U, $\ell' > \ell$. In fact d' uniquely pins down ℓ'. Next consider an arbitrary $\rho' > \rho$.

We want to show that $U_{\rho'}(\ell, d,) < U_{\rho'}(\ell', d')$. Note this will be true so long as

$$(\ell')^{\frac{\alpha}{1-\alpha}} (v\rho' - d')^{\frac{1}{1-\alpha}} > \ell^{\frac{\alpha}{1-\alpha}} (v\rho' - d)^{\frac{1}{1-\alpha}}$$

Because $U_\rho(\ell, d,) = U_\rho(\ell', d')$

$$(\ell')^{\frac{\alpha}{1-\alpha}} (v\rho - d')^{\frac{1}{1-\alpha}} = \ell^{\frac{\alpha}{1-\alpha}} (v\rho - d)^{\frac{1}{1-\alpha}}$$

Rewriting in terms of ℓ' yields

$$(I')^{\frac{\alpha}{1-\alpha}} = I^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho - d}{v\rho - d'}\right]^{\frac{1}{1-\alpha}}$$

Then

$$(I')^{\frac{\alpha}{1-\alpha}} (v\rho' - d')^{\frac{1}{1-\alpha}} = I^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho - d}{v\rho - d'}\right]^{\frac{1}{1-\alpha}} (v\rho' - d')^{\frac{1}{1-\alpha}}$$

Notice that because $(v\rho') > (v\rho')$, $d' > d$, and $\frac{1}{1-\alpha} > 0$:

$$\left[\frac{v\rho - d}{v\rho - d'}\right]^{\frac{1}{1-\alpha}} > \left[\frac{v\rho' - d}{v\rho' - d'}\right]^{\frac{1}{1-\alpha}}$$

Hence

$$\ell^{\frac{\alpha}{1-\alpha}} \left[\frac{v\rho - d}{v\rho - d'}\right]^{\frac{1}{1-\alpha}} (v\rho' - d')^{\frac{1}{1-\alpha}} > \ell^{\frac{\alpha}{1-\alpha}} (v\rho' - d)^{\frac{1}{1-\alpha}}$$

This yields the final result.

Proof of Lemma 3.3 - Suppose by contradiction that there are types $\rho \neq \rho'$ where the financing contracts the start-ups propose are identical.

We know that the effort that the start-up is willing to exert is $e = [(v\rho - d)\alpha\beta\ell^\alpha/c]^{1/1-\alpha}$.

Note that this is increasing in ρ for fixed financing contracts $\{\ell, d\}$. Call the set of all types propising the same contract $P \subset [0, \overline{\rho}]$. Note that ρ, ρ' all propose financing contract $\{\ell, d\}$. Then it must be that the expected payoff to the venture capitalist given the distribution of types in P, $G(\rho)$ is weakly positive, meaning:

$$\int_{\rho \in P} d\beta\ell^{\alpha}[(v\rho - d)\alpha\beta\ell^{\alpha}/c]^{\alpha/1-\alpha} dG(\rho) \geq \ell$$

This was derived by plugging in the effort level a start-up of type ρ is willing to exert under contracts $\{\ell, d\}$, $e = [(v\rho - d)\alpha\beta\ell^{\alpha}/c]^{1/1-\alpha}$ into the VC's payoff function. Note that this is increasing in ρ for fixed financing contracts $\{\ell, d\}$.

Rearranging we get:

$$d\beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} \int_{\rho \in P} (v\rho - d)^{\alpha/1-\alpha} dG(\rho) = \ell^{\frac{1-2\alpha}{1-\alpha}}$$

The strategy moving forward is to create a contract that a type 'above the mean' of P would take but no one below would. Now consider the types $\rho' \in P$ such that

$$(v\rho' - d)^{\alpha/1-\alpha} > \int_{\rho \in P} (v\rho - d)^{\alpha/1-\alpha} dG(\rho).$$

There are two cases: the case where there exists such a $\rho' \neq sup_{\rho}\{P\}$ and the case where the only such $\rho' = sup_{\rho}\{R\}$. Now consider the first case. Let u be the utility that the start-up of type ρ' gets from offering the pooling contracts $\{\hat{\rho}_P, \ell_P, d_P\}$. However these contracts aren't the only contracts that yield utility u. Consider the set of all financing contracts $\{\ell', d'\}$ that yield u for type ρ'. This set of contracts is all $\{\ell, d\}$ that satisfy:

$$U_{\rho'}(\ell', d') = \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] \left(\frac{\beta}{c^{\alpha}}\right)^{\frac{1}{1-\alpha}} \ell'^{\frac{\alpha}{1-\alpha}} (v\rho' - d')^{\frac{1}{1-\alpha}} = u$$

Now we pick an $\{\ell'_{\epsilon}, d'_{\epsilon}\}$ from this set such that $\ell'_{\epsilon} = \ell_R + \epsilon$ for some $\epsilon > 0$ and d' is the corresponding debt level that allows $\{\ell', d'\}$ to satisfy the above equation. Note that by the structure of the utility function, to maintain the same utility level, if ℓ increases, d must also increase. This means that $d'_\epsilon > d_P$, and d'_ϵ is increasing in ϵ. Then using our single crossing condition we find that for any $\rho < \rho'$, the start-up will strictly prefer the original funding contract $\{\ell_P, d_P\}$ to $\{\ell'_\epsilon, d'_\epsilon\}$. Then by the intuitive criterion if the venture capitalist sees $\hat{\rho}_P$ and is proposed funding contract $\{\ell', d'\}$, by the intuitive criterion, they must believe that $\rho \geq \rho'$.

The 'worst case' beliefs, $\tilde{g}$ for the venture capitalist put probability 1 on the start-up being of type ρ'. Then we claim that for sufficiently small $\epsilon > 0$, the venture capitalist's expected payoff from approving funding contract $\{\ell'_\epsilon, d'_\epsilon\}$ is greater than zero.

To show this, simply recall that the payoff to the venture capitalist from the original $\{\ell_P, d_P\}$ was strictly positive when the start-up was of type ρ'.

$$d_P \beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho' - d_P)^{\frac{\alpha}{1-\alpha}} - \ell_R^{\frac{1-2\alpha}{1-\alpha}} = \eta > 0$$

Then by the continuity of the start-up's and venture capitalist's payoffs in ℓ and d, there must be an $\epsilon > 0$ sufficiently small that

$$d'_\epsilon \beta \left[\frac{\alpha\beta}{c}\right]^{\frac{\alpha}{1-\alpha}} (v\rho - d'_\epsilon)^{\frac{\alpha}{1-\alpha}} - {\ell'_\epsilon}^{\frac{1-2\alpha}{1-\alpha}} > 0$$

So under beliefs $\tilde{g}$, the venture capitalist will accept the proposal. Hence all start-ups in P with type $\rho > \rho'$ have a profitable deviation.

For case 2 where there was only one 'above the mean' type, note that this means $sup_P \rho = \sup\{P\} \in P$. Additionally note that $sup\{P\} > \sup\{P \backslash sup\{P\}\} = \tilde{\rho}$. Then we pick some $\rho' \in (\tilde{\rho}, sup\{P\})$ (clearly $\rho' \notin P$ but the single crossing condition will still hold.) Repeat the analysis noting that a start-up of type $sup\{P\} \in P$ would have the profitable deviation and that $\tilde{g}$ implies the venture capitalist must believe for certain that the start-up is at least of type ρ'.

Thus P must be a singleton, meaning there is no pooling.

Proof of Lemma 3.4 - By contradiction - Assume that for some $\rho \in [0, \overline{\rho}]$, we have that (ℓ_ρ, d_ρ) doesn't maximize the start-up's expected utility over contracts that the venture capitalist would approve just for start-up's of type ρ. Mathematically:

$$(\ell_\rho, d_\rho) \neq \text{argmax}_{\ell,d}(v\rho - d)f(e_\rho(\ell, d), \ell) - ce_\rho(\ell, d)$$
$$s.t. \quad d * f(e_\rho(\ell, d), \ell) - \ell \geq 0. \quad (25)$$

Then let (ℓ', d') be the financing contract that does solve Equation **??**. Starting from (ℓ', d'), increase d' to d''_ϵ such that $u_\rho(\ell_\rho, d_\rho) + \epsilon = u_\rho(\ell', d''_\epsilon)$ for some small $\epsilon > 0$. Now pick a small $\delta(\epsilon) > 0$ and consider type $\tilde{\rho} = \rho - \delta(\epsilon)$. We increase both d''_ϵ by some $\eta > 0$ to $\tilde{d}$ and then pick a corresponding $\tilde{\ell}$ which makes the start-up of type $\tilde{\rho}$ exactly indifferent between the contract meant for type ρ, and $\tilde{\ell}, \tilde{d}$, so $u_{\tilde{\rho}}(\ell_\rho, d_\rho) = u_{\tilde{\rho}}(\tilde{\ell}, \tilde{d})$. Notice that for a fixed d''_ϵ, $\tilde{\ell}(\eta)$ will be strictly increasing in η, and as $\eta \to 0$, $\tilde{\ell}(\eta) \to \ell'$.

Because $\tilde{d} > d''_\epsilon$ and $u_{\tilde{\rho}}(\ell_\rho, d_\rho) = u_{\tilde{\rho}}(\tilde{\ell}, \tilde{d})$, by the single crossing property of the start-up's utility, any start-up with fewer than $\tilde{\rho}$ customers will strictly prefer the contract originally prescribed to type ρ to the new contract $(\tilde{\ell}, \tilde{d})$, and any start-up with more than $\tilde{\rho}$ customers will strictly prefer the new contract.

Since this is an equilibrium, all types below $\tilde{\rho}$ prefer their own equilibrium contracts to the contracts specified for ρ. Hence if the venture capitalist sees someone propose the contract $(\tilde{I}, \tilde{d})$, their belief, $\tilde{g}$, is that the start-up cannot be of type $\rho' < \tilde{\rho}$. Under the maximally pessimistic beliefs admittable under the intuitive criterion, $\tilde{g}$, the venture capitalist believes they are facing a start-up of type $\tilde{\rho}$ with probability 1.

Finally, since (ℓ', d') satisfied equation **??**, the venture capitalist had non-negative expected profit from (ℓ', d'). By increasing d' to d''_ϵ while holding I' fixed, the contract (ℓ', d''_ϵ) gave the venture capitalist strictly positive profit for type ρ. Then because utility is continuous in ℓ, d, and ρ for a sufficiently small ϵ, $\delta(\epsilon)$, and η, the contract $(\tilde{\ell}, \tilde{d})$ also yields positive profits for the venture capitalist if they believe the start-up is of type $\tilde{\rho}$ with probability 1.

Hence, if the Venture Capitalist sees the contract $(\tilde{I}, \tilde{d})$, they will approve it under belief $\tilde{g}$. This means the start-up of type ρ has a profitable deviation to $(\tilde{I}, \tilde{d})$. **Contradiction.**

A.2.4 Proof of Lemma 4.2

From Section A.2.3 we know that for an equilibrium to be an intuitive WPBE it must be a fully separating WPBE where the IR constraint binds that satisfies the condition set forth in Equation 25. Theorem 4.1 identified the unique WPBE that is fully separating with a binding IR constraint. Now it suffices to show that this doesn't satisfy Equation 25.

Solving the constraint in Equation 25 for ℓ and plugging it back into the maximization problem tells us that to satisfy Equation 25 d must solve

$$\max_{d \geq 0} k_1 \times d^{\frac{\alpha}{1-2\alpha}} \times (v\rho - d)^{\frac{1-\alpha}{1-2\alpha}}$$

This yields an optimal $d = \alpha(v\rho - d)$. Taking the derivative with respect to ρ yields $\frac{\partial d}{\partial \rho} = \alpha v$. This is inconsistent with Equation 16 and thus the equilibrium we found in Theorem 4.1 cannot satisfy Equation 25. As this was our only candidate intuitive WPBE, no such equilibrium exists.

A.2.5 Proof of Proposition 4.2

We know that under E^{PO} for every ρ, ℓ_ρ, d_ρ satisfy Equation 8 for $\hat{\rho}_\rho^{PO}$. First we consider ℓ'_ρ, d'_ρ which satisfy Equation 8 for $\hat{\rho} = 0$. We show that $u_\rho(\hat{\rho}_\rho^{PO}, \ell_\rho^{PO}, d_\rho^{PO}) \geq u_\rho(0, \ell'_\rho, d'_\rho) \geq u_\rho(0, \ell_\rho^D, d_\rho^D)$.

Consider ℓ'_ρ, d'_ρ:

$$\ell'_\rho = \left[\frac{\alpha\beta(1-\alpha)^\alpha}{\alpha} v\rho\right]^{\frac{1}{1-2\alpha}}, \qquad d'_\rho = \alpha v\rho.$$

The utility to type ρ of this contract is

$$U_\rho = k \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha} v\rho \right]^{\frac{\alpha}{(1-\alpha)(1-2\alpha)}} ((1-\alpha)v\rho)^{\frac{1}{1-\alpha}}.$$

Taking the derivative with respect to ρ yields

$$\frac{\partial u_\rho}{\partial \rho} = k \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha} \right]^{\frac{\alpha}{(1-\alpha)(1-2\alpha)}} (1-\alpha)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{1-2\alpha} \right) v(v\rho)^{\frac{\alpha}{1-2\alpha}}$$

Compare this to the derivative of the firm's utility with purchase orders added in

$$\frac{\partial u_\rho}{\partial \rho} = k \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha} \right]^{\frac{\alpha}{(1-\alpha)(1-2\alpha)}} (1-\alpha)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{1-2\alpha} \right) \left[v + R\frac{\partial \hat{\rho}_\rho}{\partial \rho} \right] (v\rho + R\hat{\rho}_\rho)^{\frac{\alpha}{1-2\alpha}} - R\frac{\partial \hat{\rho}_\rho}{\partial \rho}$$

Combining we find

$$k \left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha} \right]^{\frac{\alpha}{(1-\alpha)(1-2\alpha)}} (1-\alpha)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{1-2\alpha} \right) \left[\left[v + R\frac{\partial \hat{\rho}_\rho}{\partial \rho} \right] (v\rho + R\hat{\rho}_\rho)^{\frac{\alpha}{1-2\alpha}} - v(v\rho)^{\frac{\alpha}{1-2\alpha}} \right] - R\frac{\partial \hat{\rho}_\rho}{\partial \rho} > 0$$

since $\frac{\partial \hat{\rho}_\rho}{\partial \rho}$ is solved by Equation 22

With this we see that $u_\rho(\hat{\rho}_\rho, \ell_\rho, d_\rho)$ is growing faster than $u_\rho(0, \ell', d')$ for all types ρ. When $\rho = 0$ both of these utilities are equal to zero and as such we must have that $u_\rho(\hat{\rho}_\rho, \ell_\rho, d_\rho) \geq u_\rho(0, \ell', d')$ for all values of ρ.

By Equation 25, $u_\rho(0, \ell', d') \geq u_\rho(0, \ell^D_\rho, d^D_\rho)$. This yields the result.

A.3 Proofs from Section 5

A.3.1 Proof of Proposition 5.1

We use backward induction to solve for for effort. Similarly the Purchase Orders and penalty clause are already in place when the financing decision is made so they can be pinned down by Lemma 3.4.

To determine the Purchase Order Contract first note that plugging in the debt and loan from Equations **??** and 11, that R and $\hat{\rho}$ are jointly identified. This means that in terms of the firm's utiltiy, the firm is indifferent between any two combinations of R and $\hat{\rho}$ so long as their products are equal.

Next recall that the investor's expected utility is strictly increasing in the firm's type. Hence if the investor believes the firm to be of type $\rho' < \rho$ then the investor will refuse to approve any contracts that it was indifferent between approving and not for type ρ because under ρ' it is now strictly worse for the investor to approve. Starting from the maximal type an moving downward, each firm sets $\hat{\rho} = \rho$, which gives the investor a perfect signal of the firm's type without limiting the firm's contracts since R can be adjusted to suit the firm's preference for the term $R\hat{\rho}$.

Then we plug in the debt and loan from Equations **??** and 11 and set $\hat{\rho}_\rho = \rho$ and maximize with respect to R. This yields

$$R_\rho = \max\left\{\left[\frac{(1-2\alpha)c^{\frac{\alpha}{1-\alpha}}}{[(\alpha^\alpha-\alpha)\beta]^{\frac{1}{1-\alpha}}}\right]^{\frac{1-2\alpha}{2\alpha}}\left[\frac{\alpha\beta(1-\alpha)^\alpha}{c^\alpha}\right]^{\frac{1}{2\alpha}}\rho^{\frac{1-2\alpha}{2\alpha}} - v, 0\right\}. \tag{26}$$

Thus we have our equilibrium.